Need CHAIN Rule

Differentiation of Trigonometric Functions - Homework

Take the derivatives of the following functions. Identify the form of the problem and rewrite with parentheses.

1. $y = \sin 3x$

$y' = 3\cos(3x)$

2. $y = x \sin x$

$y' = x\cos x + \sin x$

3. $y = \cos\left(\frac{\pi}{2} - x\right)$

$y' = -\sin(\pi/2 - x) \cdot (-1)$

$y' = \sin(\pi/2 - x) = \cos x$

cofunction identity!

4. $y = \dfrac{\sin x}{x}$

$y' = \dfrac{x \cdot \cos x - \sin x}{x^2}$

5. $y = \dfrac{x}{\sin x} = x\csc x$

$y' = \dfrac{\sin x \cdot 1 - x \cdot \cos x}{\sin^2 x}$

$y' = \csc x - x \cdot \cot x \cdot \csc x$

6. $y = x^3 \sin^2 x = x^3 (\sin x)^2$

$y' = x^3 \cdot 2\sin x \cdot \cos x + \sin^2 x \cdot 3x^2$

$= 2x^3 \sin x\cos x + 3x^2 \sin^2 x$

$= \boxed{x^2 \sin x(2x\cos x + 3\sin x)}$ or $x^2(x\sin(2x) + 3\sin^2 x)$

7. $y = \cos 2x - \sin 3x$

$= -2\sin(2x) - 3\cos(3x)$

8. $y = \cos^4 x^4 = [\cos(x^4)]^4$

$y' = 4\cos^3(x^4) \cdot -\sin(x^4) \cdot 4x^3$

$= -16x^3 \sin(x^4)\cos^3(x^4)$

$= -8x^3 \sin(2x^4) \cdot \cos^2(x^4)$

9. $y = \sin^2 x + \cos^2 x$

$y' = 0$

10. $y = \sqrt{\sin x + 2} = (\sin x + 2)^{1/2}$

$= \frac{1}{2}(\sin x + 2)^{-1/2} \cdot \cos x$

$= \dfrac{\cos x}{2\sqrt{\sin x + 2}}$

11. $y = \tan\sqrt{3x - 1} = \tan(3x-1)^{1/2}$

$y' = \sec^2\sqrt{3x-1} \cdot \frac{1}{2}(3x-1)^{-1/2} \cdot 3$

$y' = \dfrac{3\sec^2\sqrt{3x-1}}{2\sqrt{3x-1}}$

12. $y = \sec(x^2 - 2x + 3)$

$y' = \sec(x^2 - 2x + 3)\tan(x^2 - 2x + 3) \cdot (2x - 2)$

$y' = 2(x-1)\sec(x^2 - 2x + 3)\tan(x^2 - 2x + 3)$

13) $y = \cot^4\left(\frac{x}{2}\right) = \left[\cot\left(\frac{x}{2}\right)\right]^4$

$= 4\cot^3\left(\frac{x}{2}\right) \cdot -\csc^2\left(\frac{x}{2}\right) \cdot \frac{1}{2}$

$= -2\cot^3\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)$

14) $y = \dfrac{\sin x}{1 + \cos^2 x}$

$y' = \dfrac{(1 + \cos^2 x) \cdot \cos x - \sin x(2\cos x \cdot -\sin x)}{(1 + \cos^2 x)^2}$

$y' = \dfrac{\cos x + \cos^3 x + 2\sin^2 x\cos x}{(1 + \cos^2 x)^2} = \dfrac{\cos x(1 + \cos^2 x + 2\sin^2 x)}{(1 + \cos^2 x)^2}$

$\sin^2 x + \sin^2 x$

$\boxed{\dfrac{\cos x(2 + \sin^2 x)}{(1 + \cos^2 x)^2}}$

15) $y = \sin(\cos x)$

$y' = \cos(\cos x) \cdot -\sin x = \boxed{-\sin x\cos(\cos x)}$

Find the equation of the tangent line to the following curves at the indicated point. Confirm by calculator.

16) $y = \sin x\cos x$ at $(0,0)$

See #2 previous pg

$y' = \cos(2x)$

$y'(0) = \cos(0) = 1$

$y - 0 = 1(x - 0)$

$\boxed{y = x}$

17) $y = \dfrac{2x}{\cos x}$ at $(0,0)$

$y' = \dfrac{\cos x \cdot 2 - 2x \cdot (-\sin x)}{\cos^2 x}$

$= \dfrac{2\cos x + 2x\sin x}{\cos^2 x}$

$y'(0) = \dfrac{2\cos 0 + 2 \cdot 0}{(\cos 0)^2}$

$= \dfrac{2}{1} = 2$ $\boxed{y = 2x}$

18) $y = \sin x(\sin x + \cos x)$ at $\left(\frac{\pi}{4}, 1\right)$

$y' = \sin x \cdot (\cos x - \sin x) + (\sin x + \cos x)\cos x$

$y' = \sin x\cos x - \sin^2 x + \sin x\cos x + \cos^2 x$

$y' = 2\sin x\cos x + \cos^2 x - \sin^2 x$

$y' = \sin(2x) + \cos(2x)$

$y'(\pi/4) = \sin \pi/2 + \cos \pi/2 = 1$

$\boxed{y - 1 = 1(x - \pi/4)}$ $\boxed{y = x - \pi/4 + 1}$