Lesson 27: Gravity on Inclined Planes

You need to be especially careful when you are doing problems involving gravity pulling something down a slope.

- The physics involved is considerably more complex than it might first seem, mostly because everything is tilted.
- Let's look at a standard question of gravity on an **inclined plane** (slope) to see how we would figure it out.

Example 1:**Determine** the acceleration of a 15 kg box down a 30° slope if the coefficient of friction is 0.13 on this surface.

The first thing we should do is sketch a free body diagram of the situation.

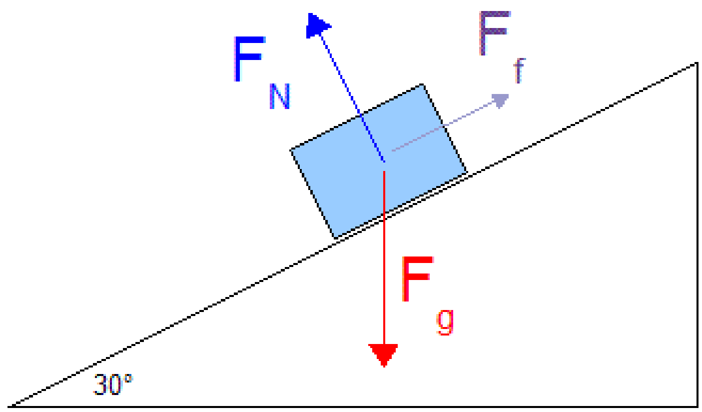

Illustration 1: Box on an inclined plane.

There are a few special things you have to notice about this diagram:

Force due to Gravity (F_g) is pointing straight down. Even though we are on a slope nothing will ever change that gravity points straight down. The only reason for this box to move down the slope will be a component of gravity's force.

Normal Force (F_N) Remember that a normal force is always perpendicular to the surface that you are on. Since this surface is slanted at a bit of an angle, the normal force will also point at a bit of an angle. In these questions $F_g \neq F_N$

Force due to Friction (F_f) will always be opposite to the direction that something is moving. In this situation the object is moving down the slope, so friction points back up the slope.

There is no **applied force** (F_a), since this would mean that there was something other than gravity actually trying to shove it down the slope. In some questions you might have an applied force also, but not in this question.

We can calculate the **force due to gravity**...

$$F_g = mg = 15(9.81) = 147.15\,N$$

We need to break F_g up into components that point down parallel to the slope ($F_{//}$) and perpendicular to the slope ($F_{\perp}$).

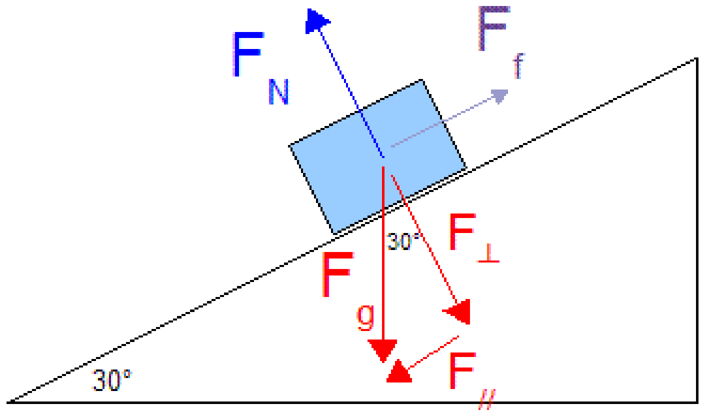

Illustration 2: F_g broken into components.

If you are wondering how I figured out that the 30°angle is at the top of the red triangle, take a look at this...

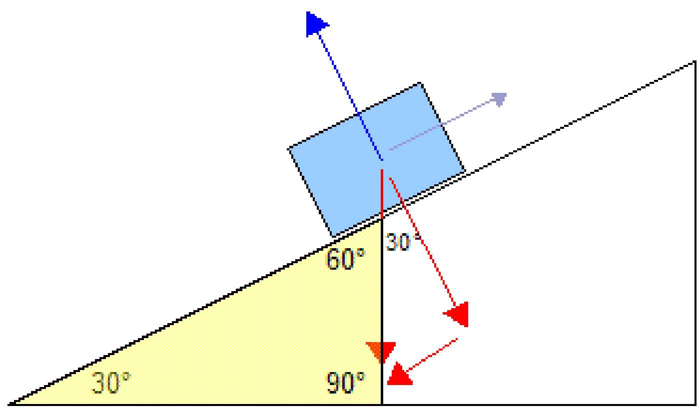

Illustration 3: Angle of slope to angle in components.

- I created the yellow triangle by just extending the line of F_g down a bit.
- Since this makes a nice right angle triangle, I know that the angle at the top of the yellow triangle must be 60° (since the angles have to add up to 180°).
- Since $F_\perp$ is perpendicular to the slope, the angle in the top of the red triangle must be 30° so that they will add up to 90°.
- Since we know F_g and an angle on the triangle, we can use basic trig to calculate the other two sides.

Determine $F_{//}$, the force pulling the box down along the slope...

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{F_{//}}{F_g}$$

$$F_{//} = \sin\theta F_g$$

$$F_{//} = \sin 30^O (147.15)$$

$$F_{//} = 73.575 N$$

Determine $F_{\perp}$...

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos\theta = \frac{F_{\perp}}{F_g}$$

$$F_{\perp} = \cos\theta(F_g)$$

$$F_{\perp} = \cos 30^{o}(147.15)$$

$$F_{\perp} = 127.436N$$

Look back at Illustration 3 and you should notice that the force perpendicular $F_{\perp}$ is equal in magnitude to the **normal force** F_N (although they point in opposite directions).

Determine the **force due to friction F_f** using the value you just got for **normal force**.

$$F_f = \mu F_N$$

$$F_f = 0.13(127.436)$$

$$F_f = 16.5667N$$

Now you know the force that is taking it down the slope, and the friction that is slowing it down. Determine the **net force F_{NET}**...

$$F_{NET} = F_{//} + F_f$$

$$F_{NET} = 73.575 + -16.5667$$

$$F_{NET} = 57.008N$$

I've made F_f negative because it is working against the $F_{//}$. One of them must be negative if the other is positive.

Now, finally we can determine the acceleration of the box...

$$F_{NET} = ma$$

$$a = \frac{F_{NET}}{m}$$

$$a = \frac{57.008}{15}$$

$$a = 3.8006 = 3.8\,m/s^2$$

Our final answer is 3.8 m/s^2 , and yes, we round it off.

Notice that in each step I had you sketch or determine something.

- I bet you can see how each of those could be a part of a multistep question like (a), (b), (c), etc.
  - In fact, I could have about 7 or 8 parts to this question.
  - If you ever do a question like that, then yes, you must round off your final answer for each step to the correct number of sig digs.
  - You should still keep the unrounded number written at least somewhere, since you should be using unrounded numbers for the questions that follow.

Re-read through this example a few times. It's long and confusing at some parts, but try to look at each individual tiny calculation. Taken in little bits, each part isn't as hard.

Homework p192 #1,3,9,14,17