

Lesson 30: Gravitational Field Strength

In the last lesson we were able to combine our two formulas for force due to gravity to get a new formula.

$$F_g = F_g$$
$$m_{\text{test}} g = \frac{G m_{\text{test}} M_e}{r^2}$$
$$g = \frac{G M_e}{r^2}$$

- The great news is that the mass (shown above as the mass of the Earth “ M_e ”) can actually be any mass. It could be the mass of the moon, Mars, an asteroid, whatever!
- In fact, we should replace **mass of the Earth** in the formula with just **mass**, since it can be any mass.

$$g = \frac{G m}{r^2}$$

g = gravitational field strength (m/s^2)
 G = Universal Gravitational Constant
 m = mass of object producing the field (kg)
 r = distance from centre of mass (m)

- This formula lets you calculate the the gravitational field strength (the acceleration due to gravity) caused by that mass ata specific distance from its centre.

Example 1: The planet Mars has a mass of $6.42\text{e}23$ kg and a radius (from its centre to the surface) of $3.38\text{e}6$ m.

- Determine** how much a 60.0 kg person would weigh on Mars.
- Compare** it to his weight on Earth.
- Determine** how heavy the 60.0 kg person would “feel” as an **apparent** mass in kilograms on Mars.

- To determine someone's weight, we first need to know the acceleration due to gravity on Mars.

$$g = \frac{G m}{r^2}$$
$$g = \frac{6.67\text{e-}11 (6.42\text{e}23)}{(3.38\text{e}6)^2}$$
$$g = 3.748240608 = 3.75 \text{ m/s}^2$$

So the person's weight will be...

$$F_g = mg$$
$$F_g = 60.0(3.748240608)$$
$$F_g = 224.8944365 = 225 \text{ N}$$

b) On Earth the person has a weight of...

$$F_g = mg$$
$$F_g = 60.0(9.81)$$
$$F_g = 588.6 = 589 \text{ N}$$

Probably the easiest way to compare the person's weight on Mars and the Earth is to find the ratio of the two.

$$\frac{F_{g\text{earth}}}{F_{g\text{Mars}}} = \frac{589}{225} = 2.6172279 = 2.62$$

Since this is a ratio, it has no units (the Newtons canceled each other out). It simply means that you weigh 2.62 times more on the Earth as compared to Mars.

c) The reason I asked for the person's **apparent** mass is because I want to know how heavy the person thinks he feels in kilograms. In reality, **the true mass of a person never changes**. I am asking how heavy he feels based on the facts that he *feels* lighter on Mars, and that they are used to the effects of gravity on Earth. We will take the person's weight on Mars and the gravity of Earth to find out the apparent mass.

$$F_g = mg$$
$$m = \frac{F_g}{g}$$
$$m = \frac{224.8944365}{9.81}$$
$$m = 22.925019 = 22.9 \text{ kg}$$

Keep in mind that the person's mass is still really 60.0 kg. He just feels like he is only 22.9 kg because he is on Mars.

The Elevator Question

The concept of gravity becomes a bit more complicated when you examine a complex system like an elevator going up and down.

- It might sound strange to call an elevator complex, but it really does make a challenging problem.
- How do you think you would solve a question that asks you about your weight as an elevator accelerates up or down?

Going up...

When the elevator is accelerating up, what would happen to your weight?

- Have you ever noticed that when an elevator first starts to move up, you feel as though you are being pushed down a little?
 - This is because you can feel the elevator's acceleration. The elevator is pushing you up, so (according to Newton's Third Law) you push down against the floor.
- It basically makes you feel a bit heavier for a moment.
 - A scale would show this as an increase in your weight (temporarily). If the scale shows kilograms, it would show your **apparent** mass as being bigger than your true mass.

Going down...

What would happen to your weight if the elevator started to accelerate down?

- You would feel the elevator drop out underneath you.
 - If it really dropped out underneath you, it would feel just like being on the "Space Shot" at Galaxyland as it is falling ... you'd feel weightless!
- You feel this way because as the elevator accelerates down (away from you) it is not pushing up against you as hard as it was before. Since the floor is not pushing as hard up against you, you are not pushing as hard down against the floor (Newton's Third Law again).
 - A scale would show your weight as being less. If the scale shows kilograms, it would show your **apparent** mass as being smaller than your true mass.

Let's look at how we would actually figure out some numbers for this type of question by looking at an example.

- Keep the following in the back of your mind. A regular scale that you buy in a regular store is made to measure things in kilograms, and it is built for Earth's regular gravity of 9.81m/s^2 . You'll see why this is important later.

Example 2: You are standing on a scale in an elevator. You have a mass of 75kg. Determine what a scale would show as your **apparent** mass (in kilograms) if...

a) the elevator starts to accelerate upwards at 3.0m/s^2 .

We have three parts in the formula that we will have to use:

F_{NET} = the overall force acting on the person causing acceleration upwards.

F_g = the force due to gravity pulling the person down.

F_N = the force of the scale on the floor pushing up against the person.

We should mostly be concerned with what the normal force is, since however hard the scale has to push the person upwards will show up as a reading on the scale. We know the force due to gravity, since that's just the person's weight. The net force is just overall what is happening to the person.

$$\begin{aligned}F_{\text{NET}} &= F_g + F_N \\ ma &= mg + F_N \\ ma - mg &= F_N \\ m(a - g) &= F_N \\ 75[3.0 - (-9.81)] &= F_N \\ 75[12.81] &= F_N \\ F_N &= 960.75 \\ F_N &= 9.6 \times 10^2 \text{ N}\end{aligned}$$

We start off with a standard net force formula. We do a quick substitution, since $F_{\text{NET}} = ma$ and $F_g = mg$. Next step is to move "mg" to the other side. Since both formulas on the left side have "m" we can factor it out. Now start putting in the numbers, watching out for the directions of the accelerations. This lets us calculate the normal force. As hard as the scale is pushing the person up, the person must be pushing down just as hard which tells us the weight the scale will show in Newtons.

The question asked about the **apparent** mass, not the weight of the person.

- I can change this into a reading in kilograms by remembering that the scale we're using has no idea what is going on... it was originally calibrated to be sitting in someone's bathroom where gravity is a nice constant 9.81m/s^2 .
- This is **not** the true mass of the person, since mass never really changes.

$$\begin{aligned}F_N &= mg \\ m &= \frac{F_N}{g} \\ m &= \frac{960.75}{9.81} \\ m &= 97.93577982 = 98\text{ kg}\end{aligned}$$

So a regular scale shows an **apparent** mass of 98 kg!

b) the elevator starts to accelerate downwards at 4.0m/s^2 .

We'll handle this part of the question the same way.

$$\begin{aligned}F_{NET} &= F_g + F_N \\ ma &= mg + F_N \\ ma - mg &= F_N \\ m(a - g) &= F_N \\ 75[-4.0 - (-9.81)] &= F_N \\ 75[5.81] &= F_N \\ F_N &= 435.75 = 4.4\text{e}2\text{ N}\end{aligned}$$

Since the scale is pushing up against the person with $4.4\text{e}2\text{ N}$ of force (the weight that shows on the scale in Newtons), the person's apparent mass will be...

$$\begin{aligned}F_N &= mg \\ m &= \frac{F_N}{g} \\ m &= \frac{435.75}{9.81} \\ m &= 44.418960 = 44\text{ kg}\end{aligned}$$

So a regular scale shows your apparent mass as 44 kg.

Homework

p.219 #1-3

p.225 #1

p.231 #1, 3-6, 8, 9