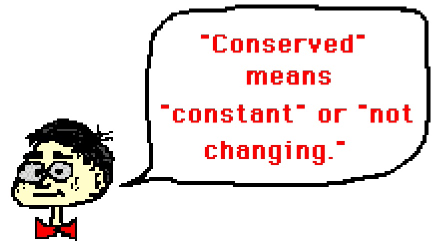
The Law of Momentum Conservation

Conservation Principle

One of the most powerful laws in physics is the law of momentum conservation. The law of momentum conservation can be stated as follows.

For a collision occurring between object 1 and object 2 in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision. That is, the momentum lost by object 1 is equal to the momentum gained by object 2.

The above statement tells us that the total momentum of a collection of objects (a *system*) is *conserved* - that is, the total amount of momentum is a constant or unchanging value. This law of momentum conservation will be the focus of the remainder of Lesson 2. To understand the basis of momentum conservation, let's begin with a short logical proof.



Consider a collision between two objects - object 1 and object 2. For such a collision, the forces acting between the two objects are equal in magnitude and opposite in direction (Newton's third law). This statement can be expressed in equation form as follows.

$$F_1 = - F_2$$

The forces are equal in magnitude and opposite in direction.

The forces act between the two objects for a given amount of time. In some cases, the time is long; in other cases the time is short. Regardless of how long the time is, it can be said that the time that the force acts upon object 1 is equal to the time that the force acts upon object 2. This is merely logical. Forces result from interactions (or contact) between two objects. If object 1 contacts object 2 for 0.050 seconds, then object 2 must be contacting object 1 for the same amount of time (0.050 seconds). As an equation, this can be stated as

$$t_1 = t_2$$

Since the forces between the two objects are equal in magnitude and opposite in direction, and since the times for which these forces act are equal in magnitude, it follows that the impulses experienced by the two objects are also equal in magnitude and opposite in direction. As an equation, this can be stated as

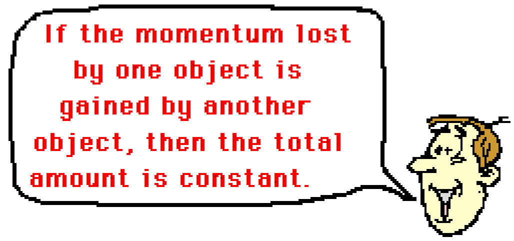
$$F_1 * t_1 = - F_2 * t_2$$

The impulses are equal in magnitude and opposite in direction.

But the impulse experienced by an object is equal to the change in momentum of that object (the impulse-momentum change theorem). Thus, since each object experiences equal and opposite impulses, it follows logically that they must also experience equal and opposite momentum changes. As an equation, this can be stated as

$$m_1 * \Delta v_1 = - m_2 * \Delta v_2$$

The momentum changes are equal in magnitude and opposite in direction.



The above equation is one statement of the law of momentum conservation. In a collision, the momentum change of object 1 is equal to and opposite of the momentum change of object 2. That is, the momentum lost by object 1 is equal to the momentum gained by object 2. In most collisions between two objects, one object slows down and loses momentum while the other object speeds up and gains momentum. If object 1 loses 75 units of momentum, then object 2 gains 75 units of momentum. Yet, the total momentum of the two objects (object 1 plus object 2) is the same before the collision as it is after the collision. The total momentum of *the system* (the collection of two objects) is conserved.

A useful analogy for understanding momentum conservation involves a money transaction between two people. Let's refer to the two people as Jack and Jill. Suppose that we were to check the pockets of Jack and Jill before and after the money transaction in order to determine the amount of money that each possesses. Prior to the transaction, Jack possesses $100 and Jill possesses $100. The total amount of money of the two people before the transaction is $200. During the transaction, Jack pays Jill $50 for the given item being bought. There is a transfer of $50 from Jack's pocket to Jill's pocket. Jack has lost $50 and Jill has gained $50. The money lost by Jack is equal to the money gained by Jill. After the transaction, Jack now has $50 in his pocket and Jill has $150 in her pocket. Yet, the total amount of money of the two people after the transaction is $200. The total amount of money (Jack's money plus Jill's money) before the transaction is equal to the total amount of money after the transaction. It could be said that the total amount of money of *the system* (the collection of two people) is conserved. It is the same before as it is after the transaction.

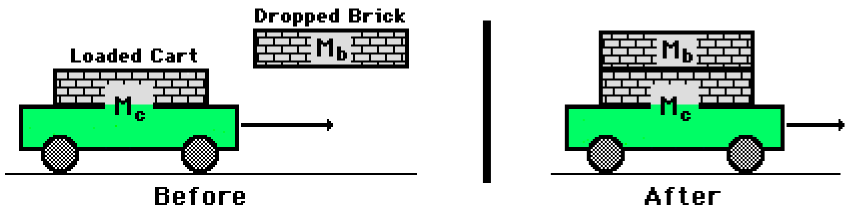
A useful means of depicting the transfer and the conservation of money between Jack and Jill is by means of a table.

Money Conservation in a Financial Interaction

	Before	After	Change
Jack	$100	$50	-$50
Jill	$100	$150	+$50
Total	$200	$200	

The table shows the amount of money possessed by the two individuals before and after the interaction. It also shows the total amount of money before and after the interaction. Note that the total amount of money ($200) is the same before and after the interaction - it is conserved. Finally, the table shows the change in the amount of money possessed by the two individuals. Note that the change in Jack's money account (-$50) is equal to and opposite of the change in Jill's money account (+$50).

For any collision occurring in an isolated system, momentum is conserved. The total amount of momentum of the collection of objects in the system is the same before the collision as after the collision. A common physics lab involves the dropping of a brick upon a cart in motion.

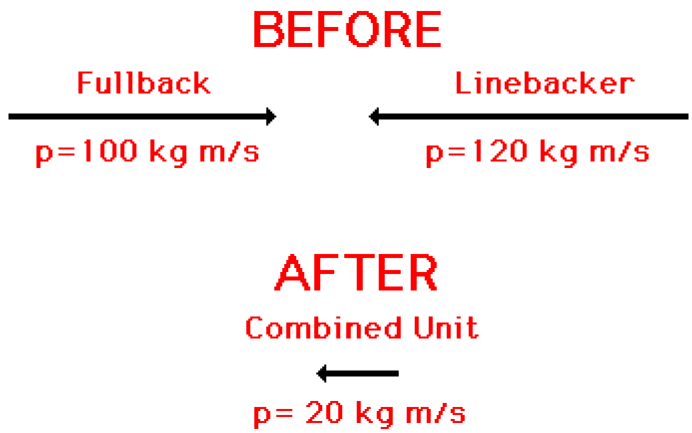


The dropped brick is at rest and begins with zero momentum. The loaded cart (a cart with a brick on it) is in motion with considerable momentum. The actual momentum of the loaded cart can be determined using the velocity (often determined by a ticker tape analysis) and the mass. The total amount of momentum is the sum of the dropped brick's momentum (0 units) and the loaded cart's momentum. After the collision, the momenta of the two separate objects (dropped brick and loaded cart) can be determined from their measured mass and their velocity (often found from a ticker tape analysis). If momentum is conserved during the collision, then the sum of the dropped brick's and loaded cart's momentum after the collision should be the same as before the collision. The momentum lost by the loaded cart should equal (or approximately equal) the momentum gained by the dropped brick. Momentum data for the interaction between the dropped brick and the loaded cart could be depicted in a table similar to the money table above.

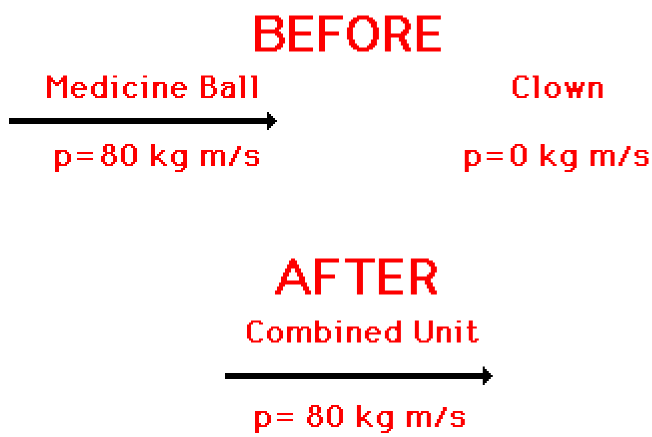
	Before Collision Momentum	**After Collision Momentum**	**Change in Momentum**
Dropped Brick	0 units	14 units	+14 units
Loaded Cart	45 units	31 units	-14 units
Total	45 units	45 units	

Note that the loaded cart lost 14 units of momentum and the dropped brick gained 14 units of momentum. Note also that the total momentum of the system (45 units) was the same before the collision as it was after the collision.

Collisions commonly occur in contact sports (such as football) and racket and bat sports (such as baseball, golf, tennis, etc.). Consider a collision in football between a fullback and a linebacker during a *goal-line stand*. The fullback plunges across the goal line and collides in midair with the linebacker. The linebacker and fullback hold each other and travel together after the collision. The fullback possesses a momentum of 100 kg*m/s, East before the collision and the linebacker possesses a momentum of 120 kg*m/s, West before the collision. The total momentum of the system before the collision is 20 kg*m/s, West (review the section on adding vectors if necessary). Therefore, the total momentum of the system after the collision must also be 20 kg*m/s, West. The fullback and the linebacker move together as a single unit after the collision with a combined momentum of 20 kg*m/s. Momentum is conserved in the collision. A vector diagram can be used to represent this principle of momentum conservation; such a diagram uses an arrow to represent the magnitude and direction of the momentum vector for the individual objects before the collision and the combined momentum after the collision.



Now suppose that a medicine ball is thrown to a clown who is at rest upon the ice; the clown catches the medicine ball and glides together with the ball across the ice. The momentum of the medicine ball is 80 kg*m/s before the collision. The momentum of the clown is 0 m/s before the collision. The total momentum of the system before the collision is 80 kg*m/s. Therefore, the total momentum of the system after the collision must also be 80 kg*m/s. The clown and the medicine ball move together as a single unit after the collision with a combined momentum of 80 kg*m/s. Momentum is conserved in the collision.



Momentum is conserved for any interaction between two objects occurring in an isolated system. This conservation of momentum can be observed by a total system momentum analysis or by a momentum change analysis. Useful means of representing such analyses include a momentum table and a vector diagram. Later in Lesson 2, we will use the momentum conservation principle to solve problems in which the after-collision velocity of objects is predicted.

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