

Name : _____

For numbers 1-3, use the *sum/difference* formulas to evaluate:

150 + 45

$$1) \sin(195^\circ) = \sin 150 \cos 45 + \cos 150 \sin 45$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$2) \cos\left(\frac{5\pi}{12}\right) = \cos \frac{\pi}{4} \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \sin \frac{\pi}{6}$$

$$\frac{3\pi}{12} \text{ r } \frac{2\pi}{12}$$

$$\frac{\pi}{4} + \frac{\pi}{6}$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$3) \tan\left(\frac{-7\pi}{12}\right) = \frac{\tan^{-\pi/4} - \tan^{\pi/3}}{1 + \tan(-\pi/4)\tan(\pi/3)} = \frac{-1 - \sqrt{3}}{1 + (-1)(\sqrt{3})} = \frac{-1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$\frac{-3\pi}{12} - \frac{4\pi}{12}$$

$$\downarrow \quad \downarrow$$

$$-\frac{\pi}{4} \quad - \quad \frac{\pi}{3}$$

For numbers 4 and 5, use the *half angle* formulas to evaluate:

b/c 15° in Q1

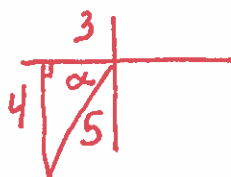
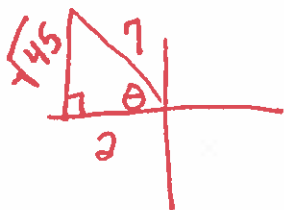
$$4) \sin(15^\circ) = \sqrt{\frac{1 - \cos 30}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

b/c 112.5 in Q2

$$5) \cos(112.5^\circ) = \sqrt{\frac{1 + \cos 225}{2}} = \sqrt{\frac{1 + \frac{-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

6) $\cos \theta = -\frac{2}{7}$ $\frac{\pi}{2} < \theta < \pi$

$\tan \alpha = \frac{4}{3}$ $\pi < \alpha < \frac{3\pi}{2}$



a) $\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos \theta}{2}}$
 $= \sqrt{\frac{1 - (-2/7)}{2}} = \frac{7/7 + 2/7}{2}$
 $= \frac{9/7}{2} = \frac{9}{14}$

b) $\cos(2\alpha) = 2 \cos^2 \alpha - 1$
 $= 2 \left(-\frac{3}{5}\right)^2 - 1$
 $= 2 \left(\frac{9}{25}\right) - 1$
 $= \frac{18}{25} - \frac{25}{25} = -\frac{7}{25}$

c) $\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$\frac{2 \left(-\frac{\sqrt{45}}{2}\right)}{1 - \left(-\frac{\sqrt{45}}{2}\right)^2} = \frac{-\sqrt{45}}{1 - \frac{45}{4}}$
 $= \frac{-\sqrt{45}}{-41/4} = \frac{4\sqrt{45}}{41} = \frac{12\sqrt{5}}{41}$

d) $\cos(\alpha - \theta) =$

$\cos \alpha \cos \theta + \sin \alpha \sin \theta$
 $\left(-\frac{3}{5}\right) \left(-\frac{2}{7}\right) + \left(-\frac{4}{5}\right) \left(\frac{\sqrt{45}}{7}\right)$
 $= \frac{6}{35} + \frac{-4\sqrt{45}}{35} = \frac{6 - 4\sqrt{45}}{35}$
 $= \frac{6 - 12\sqrt{5}}{35}$

e) $\sin(\theta + \alpha) =$

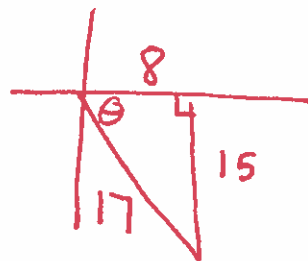
$\sin \theta \cos \alpha + \cos \theta \sin \alpha$
 $\left(\frac{\sqrt{45}}{7}\right) \left(-\frac{3}{5}\right) + \left(-\frac{2}{7}\right) \left(-\frac{4}{5}\right)$
 $= \frac{-3\sqrt{45}}{35} + \frac{8}{35} = \frac{-9\sqrt{5} + 8}{35}$

f) $\tan(\alpha + \theta) =$

$\frac{\tan \alpha \tan \theta}{1 - \tan \alpha \tan \theta}$
 $= \frac{\frac{4}{3} + \frac{-\sqrt{45}}{2}}{1 - \left(\frac{4}{3}\right) \left(-\frac{\sqrt{45}}{2}\right)} = \frac{\frac{8}{6} - \frac{3\sqrt{45}}{6}}{\frac{6}{6} + \frac{4\sqrt{45}}{6}}$
 $= \frac{8 - 3\sqrt{45}}{6 + 4\sqrt{45}} = \frac{8 - 9\sqrt{5}}{6 + 12\sqrt{5}}$

$$\cos \theta = \frac{8}{17}$$

$$3\pi/2 < \theta < 2\pi$$



1. $\sec 2\theta =$
Find $\cos 2\theta$
then flip

$$2\cos^2 \theta - 1 = 2\left(\frac{8}{17}\right)^2 - 1 = \frac{128}{289} - \frac{289}{289} = \frac{-161}{289}$$

$$17^2 = 8^2 + b^2$$

$$225 = b^2$$

$$15 = b$$

$$\frac{289}{161}$$

2. $\cot 2\theta =$
Find $\tan 2\theta$
then flip

$$\frac{2\tan \theta}{1 - \tan^2 \theta} = \frac{2(-15/8)}{1 - (-15/8)^2} = \frac{-30/8}{1 - 225/64} = \frac{-30/8}{-161/64} = \frac{-30 \times 64}{8 \times -161} = \frac{-240}{-161} = \frac{240}{161}$$

3. $\csc \frac{\theta}{2} =$
Find $\sin \frac{\theta}{2}$
then Flip

$$+\sqrt{\frac{1 - (\cos \theta)}{2}} = +\sqrt{\frac{1 - (8/17)}{2}} = +\sqrt{\frac{17/17 - 8/17}{2}} = +\sqrt{\frac{9/17}{2}} = +\sqrt{\frac{9}{34}}$$

$$= \frac{-240}{-161} = \frac{240}{161}$$

$$\frac{161}{240}$$

θ b/w $270 + 360$ so $1/2 \theta$ b/w $135 + 180 \rightarrow Q2 \rightarrow \sin(+)$

$$+\sqrt{\frac{34}{9}} = \frac{\sqrt{34}}{3}$$

$$4. \csc 2\theta = 2(\sin \theta)(\cos \theta)$$

Find $\sin 2\theta$
then Flip

$$\frac{2(-15/17)(8/17)}{1} = \frac{-240}{289}$$

$$\frac{289}{240}$$

$$5. \sec \frac{\theta}{2} =$$

Find $\cos \frac{\theta}{2}$
then flip

$$-\sqrt{\frac{1 + \cos \theta}{2}} = -\sqrt{\frac{1 + 8/17}{2}} = -\sqrt{\frac{17/17 + 8/17}{2}} = -\sqrt{\frac{25/17}{2}} = -\sqrt{\frac{25}{34}}$$

$$= -\sqrt{\frac{34}{25}}$$

$$6. \cot \frac{\theta}{2} =$$

Find $\tan \frac{\theta}{2}$
then flip

$$\frac{1 - \cos \theta}{\sin \theta} = \frac{1 - 8/17}{-15/17} = \frac{9/17 \times -17}{-15} = \frac{-9}{-15} = \frac{3}{5}$$

$$\frac{-15}{9}$$

$$\frac{3}{5}$$

Solve each equation for $0 \leq \theta < 360$.

7) $2\sin 2\theta = -\sqrt{2}\sin \theta + \sin 2\theta$

$-2\sin 2\theta$ $-2\sin 2\theta$

$0 = -\sqrt{2}\sin \theta - \sin 2\theta$

↓

$0 = -\sqrt{2}\sin \theta - 2\sin \theta \cos \theta$

$0 = \sin \theta (-\sqrt{2} - 2\cos \theta)$

↙

$\sin \theta = 0$

$\theta = 0^\circ, 180^\circ$

$-\sqrt{2} - 2\cos \theta = 0$

$-2\cos \theta = \sqrt{2}$

$\cos \theta = -\frac{\sqrt{2}}{2}$

$\theta = 135^\circ, 225^\circ$

8) $-\sqrt{3}\sin \theta = -\sin 2\theta$

$+\sin 2\theta$

$-\sqrt{3}\sin \theta + \sin 2\theta = 0$

↓

$-\sqrt{3}\sin \theta + 2\sin \theta \cos \theta = 0$

$\sin \theta (-\sqrt{3} + 2\cos \theta) = 0$

↓

$\sin \theta = 0$

$\theta = 0^\circ, 180^\circ$

$-\sqrt{3} + 2\cos \theta = 0$

$2\cos \theta = \sqrt{3}$

$\cos \theta = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ, 330^\circ$

9) $\cos^2 \theta - 3\sin^2 2\theta = -2\sin^2 2\theta$

$+2\sin^2 2\theta + 2\sin^2 2\theta$

$\cos^2 \theta - \sin^2 2\theta = 0$

$\cos^2 \theta - (\sin 2\theta)(\sin 2\theta) = 0$

↓

↓

$\cos^2 \theta - (2\sin \theta \cos \theta)(2\sin \theta \cos \theta)$

$\cos^2 \theta - 4\sin^2 \theta \cos^2 \theta = 0$

$\cos^2 \theta (1 - 4\sin^2 \theta) = 0$

↙

$\sqrt{\cos^2 \theta} = \sqrt{0}$

$\cos \theta = 0$

$\theta = 90^\circ, 270^\circ$

$1 - 4\sin^2 \theta = 0$

$1 = 4\sin^2 \theta$

$\pm \sqrt{\frac{1}{4}} = \sqrt{\sin^2 \theta}$

$\pm \frac{1}{2} = \sin \theta$

$\theta = 30^\circ, 150^\circ$
 $210^\circ, 330^\circ$

10) $3\sin 2\theta = -2\sin \theta + 4\sin 2\theta$

$-3\sin 2\theta$

$-3\sin 2\theta$

$0 = -2\sin \theta + \sin 2\theta$

↓

$0 = -2\sin \theta + 2\sin \theta \cos \theta$

$0 = -2\sin \theta (1 - \cos \theta)$

↓

↓

$-2\sin \theta = 0$

$\sin \theta = 0$

$\theta = 0^\circ, 180^\circ$

$1 - \cos \theta = 0$

$1 = \cos \theta$

$\theta = 0^\circ$