Chapter 5.1-2 Review

More practice problems…in order of difficulty.

1. $\dfrac{\cos x}{1-\sin^2 x} = \sec x$ $\quad \dfrac{\cos x}{\cos^2 x} = \dfrac{1}{\cos x} = \sec x$

2. $\sin x + \sin x \cot^2 x = \csc x$ $\quad \sin x\left(1+\cot^2 x\right)$ $\quad \sin x\left(\csc^2 x\right) = \sin x \cdot \dfrac{1}{\sin^2 x}$ $\quad = \dfrac{1}{\sin x} = \csc x$

3. $3\tan^4 x - \tan^2 x - 4$ $\quad (3\tan^2 x - 4)(\tan^2 x + 1)$ $\quad (3\tan^2 x - 4)(\sec^2 x)$

4. $\sin\left(\dfrac{\pi}{2} - x\right)\cos(-x) = \cos^2 x$ $\quad \cos x \cdot \cos x = \cos^2 x$

5. $\dfrac{\cos x + \tan x}{\sin x} = \cot x + \sec x$ $\quad \dfrac{\cos x}{\sin x} + \dfrac{\tan x}{\sin x} = \cot x + \dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\sin x}$ $\quad = \cot x + \dfrac{1}{\cos x}$ $\quad \cot x + \sec x$

6. $\dfrac{\sec^4 x - 1}{\sec^4 x + \sec^2 x} = \sin^2 x$ $\quad \dfrac{(\sec^2 x - 1)(\sec^2 x + 1)}{\sec^2 x(\sec^2 x + 1)} = \dfrac{\tan^2 x}{\sec^2 x} = \dfrac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$ $\quad = \dfrac{\sin^2 x}{\cos^2 x}\cdot\dfrac{\cos^2 x}{1}$ $\quad = \sin^2 x$

7. $\dfrac{\sin x}{1+\cos x} = \csc x - \cot x$ $\quad \dfrac{\sin x}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x} = \dfrac{\sin x(1-\cos x)}{1-\cos^2 x} = \dfrac{\sin x(1-\cos x)}{\sin^2 x}$ $\quad = \dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}$ $\quad \csc x - \cot x$

$$\frac{(\csc^2 x - 1)(\csc^2 x + 1)}{\cot^2 x} = \frac{\cot^2 x\,(\csc^2 x + 1)}{\cot^2 x}$$

8. $2 + \cot^2 x = \dfrac{\csc^4 x - 1}{\cot^2 x}$

$= \cot^2 x + 1 + 1$

$= 2 + \cot^2 x$

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9. $\dfrac{1+\cos x}{\sin x} + \dfrac{\sin x}{1+\cos x} = 2\csc x$

$$\frac{(1+\cos x)(1+\cos x) + (\sin x)(\sin x)}{\sin x(1+\cos x)}$$

$$= \frac{1 + 2\cos x + \cos^2 x + \sin^2 x}{\sin x(1+\cos x)}$$

$$= \frac{1 + 2\cos x + 1}{\sin x(1+\cos x)}$$

$$= \frac{2(1+\cos x)}{\sin x(1+\cos x)}$$

$$= \frac{2}{\sin x} = 2\csc x$$

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10. $\dfrac{\cot^2 x}{\csc^2 x - \csc x} = 1 + \sin x$

$$\frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x} - \frac{1}{\sin x}} = \frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1 - \sin x}{\sin^2 x}}$$

$$= \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1 - \sin x}$$

$$= \frac{\cos^2 x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x}$$

$$= \frac{\cos^2 x\,(1 + \sin x)}{1 - \sin^2 x} = \frac{\cos^2 x\,(1 + \sin x)}{\cos^2 x}$$

$$= 1 + \sin x$$

* See attached sheet... much easier way!

identity!

$$\frac{\cot^2 x}{\csc^2 x - \csc x}$$

#10

factor out GCF

$$\frac{\csc^2 x - 1}{\csc x(\csc x - 1)} = \frac{(\csc x + 1)(\csc x - 1)}{\csc x(\csc x - 1)}$$

$$= \frac{\csc x + 1}{\csc x}$$

much easier.

$$= \frac{\csc x}{\csc x} + \frac{1}{\csc x}$$

$$= 1 + \sin x$$