

# Chap 6 Review

## Day 1

1.  $f(x) = 22x^9 - 9x^5 + 25x^3 + 16x^2 - x$

Deg: 9
Lead Coeff: 22
# of terms: 5

7. 2	-2	0	7	0	-3	-9
	-4	-8	-2	-4	-14	
	-2	-4	-1	-2	-7	-23

2.  $(x-2)^2(3x^3 - 2x^2 + 1)$

$(x^2 - 4x + 4)(3x^3 - 2x^2 + 1)$

$$\begin{array}{r}
 3x^5 - 2x^4 + \phantom{0}x^2 \\
 -12x^4 + 8x^3 - 4x \\
 +12x^3 - 8x^2 + 4 \\
 \hline
 3x^5 - 14x^4 + 20x^3 - 7x^2 - 4x + 4
 \end{array}$$

$P(2) = -23$

8-11 → See graphs

$3x^5 - 14x^4 + 20x^3 - 7x^2 - 4x + 4$

12.  $f(-x) - 5$   
 $= 3 + 2(-x) - (-x)^2 - 5$   
 $= 3 - 2x - x^2 - 5$   
 $= -2 - 2x - x^2$

3.  $(2x-1)^4$

$$\begin{array}{r}
 1 \quad 3 \quad 3 \quad 1 \\
 1 \quad 4 \quad 6 \quad 4 \quad 1 \\
 (2x)^4 + 4(2x)^3(-1) + 6(2x)^2(-1)^2 + 4(2x)(-1)^3 + (-1)^4 \\
 \hline
 16x^4 - 32x^3 + 24x^2 - 8x + 1
 \end{array}$$

4.  $x^2 - 4x + 3 + \frac{3}{2x-1}$

$$\begin{array}{r}
 2x-1 \overline{) 2x^3 - 9x^2 + 10x} \\
 \underline{-2x^3 + x^2} \phantom{+ 10x} \\
 -8x^2 + 10x \\
 \underline{+8x^2 - 4x} \\
 6x \\
 \underline{-6x + 3} \\
 3
 \end{array}$$

13.  $-f(x-3) + 2$   
 $= -(3 + 2(x-3) - (x-3)^2) + 2$   
 $= -(3 + 2x - 6 - x^2 + 6x - 9) + 2$   
 $= -(-x^2 + 8x - 12) + 2$   
 $= x^2 - 8x + 12 + 2$   
 $= x^2 - 8x + 14$

5.  $-2 \overline{) 2 \ 0 \ 0 \ 0 \ -1}$

2	0	0	0	-1
-4	8	-16	32	
2	-4	8	-16	31

$= 2x - 4x + 8x - 16 + \frac{31}{x+2}$

## Day 2

6.  $\frac{4x^4}{2x} - \frac{3x^3}{2x} + \frac{10x}{2x} = 2x^3 - \frac{3}{2}x^2 + 5$

1.  $x = -2, x = 5 \pm \sqrt{3}$   
 $x+2=0, (x-5)^2 = (\pm\sqrt{3})^2$   
 $x^2 - 10x + 25 = 3$   
 $x^2 - 10x + 22 = 0$

$y = (x+2)(x^2 - 10x + 22)$   
 $y = x^3 - 10x^2 + 22x + 2x^2 - 20x + 44$   
 $y = x^3 - 8x^2 + 2x + 44$

2.  $x = 2 \pm i$ ,  $x = 3 \pm \sqrt{2}$

$(x-2)^2 = (\pm i)^2$

$x^2 - 4x + 4 = -1$

$x^2 - 4x + 5 = 0$

$(x-3)^2 = (\pm\sqrt{2})^2$

$x^2 - 6x + 9 = 2$

$x^2 - 6x + 7 = 0$

$(x^2 - 4x + 5)(x^2 - 6x + 7) = 0$

$y = x^4 - 6x^3 + 7x^2 - 4x^3 + 24x^2 - 28x$

$y = x^4 - 10x^3 + 36x^2 - 58x + 35$

	$x^5$					
6. 2	1	-5	-1	41	-72	36
		2	-6	-14	54	-36
	1	-3	-7	27	-18	0

2	1	-3	-7	27	-18
		2	-2	-18	18
	1	-1	-9	9	0

$x^3$

$= (x-2)^2 (x^3 - x^2 - 9x + 9)$   
 $= (x-2)^2 (x^2(x-1) - 9(x-1))$   
 $= (x-2)^2 (x-1)(x^2 - 9)$   
 $= (x-2)^2 (x-1)(x-3)(x+3)$

3. -2	3	11	2	-16
		-6	-10	16
	3	5	-8	0

$x = 2, 1, 3, -3$

$(x+2)(3x^2 + 5x - 8) = 0$

$(x+2)(3x+8)(x-1) = 0$

$x = -2, -8/3, 1$

7-8 → See graphs.

4.  $f(x) = x(4x^4 + 5x^2 - 9)$   
 $= x(4x^2 + 9)(x^2 - 1)$   
 $= x(x-1)(x+1)(4x^2 + 9) = 0$

$4x^2 + 9 = 0$

$4x^2 = -9$

$x^2 = -9/4$

$x = \pm \frac{3}{2}i$

$x = 0, 1, -1, \pm \frac{3}{2}i$

5.  $f(x) = -3x^2(x-10) + 5(x-10)$   
 $= (x-10)(-3x^2 + 5) = 0$

$x = 10, \pm \frac{\sqrt{15}}{3}$

$-3x^2 = -5$

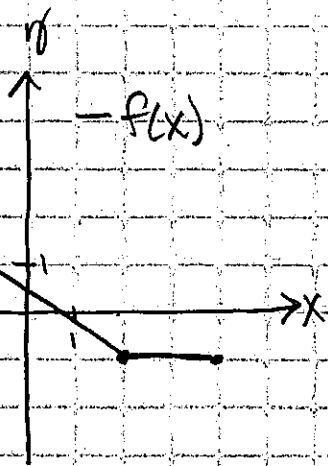
$x^2 = 5/3$

$x = \pm \sqrt{5/3}$

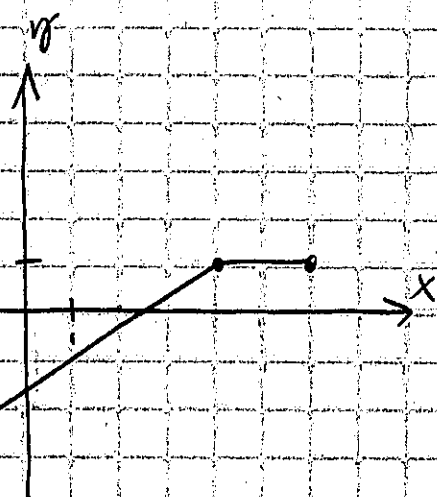
$x = \pm \frac{\sqrt{15}}{3}$

Day 1

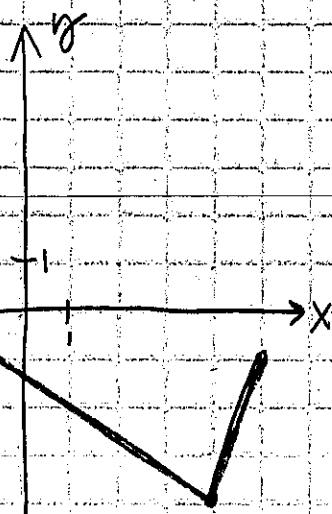
8.



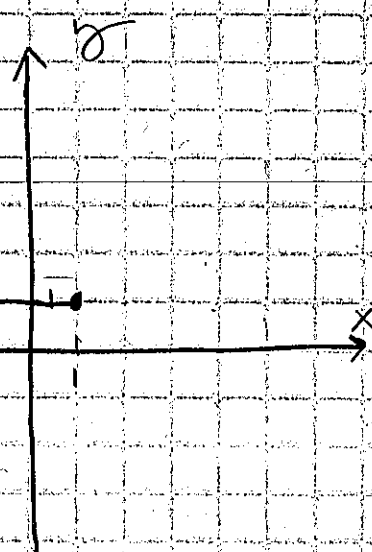
9.  $f(x-2)$



10.  $f(-x) - 1$



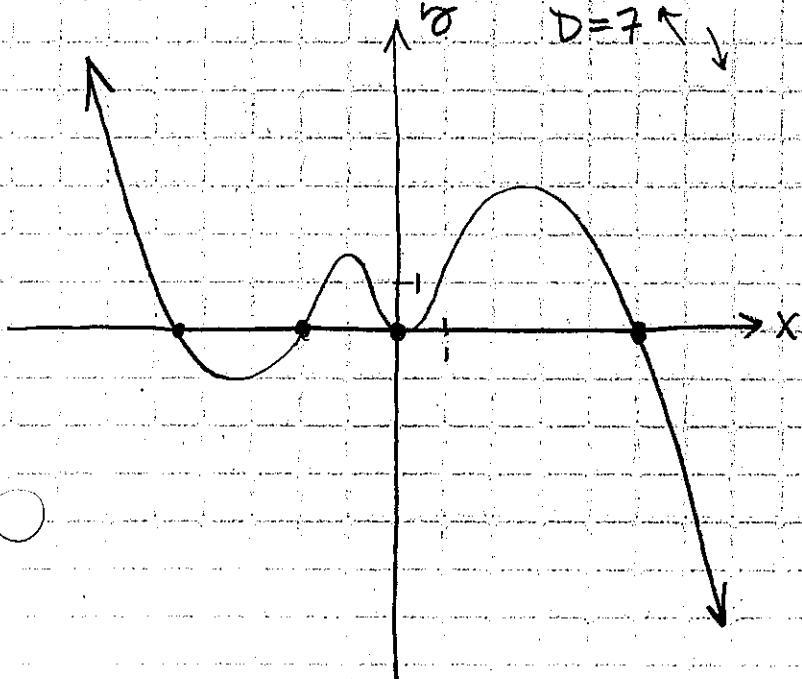
11.  $2 - f(x+3)$



Day 2

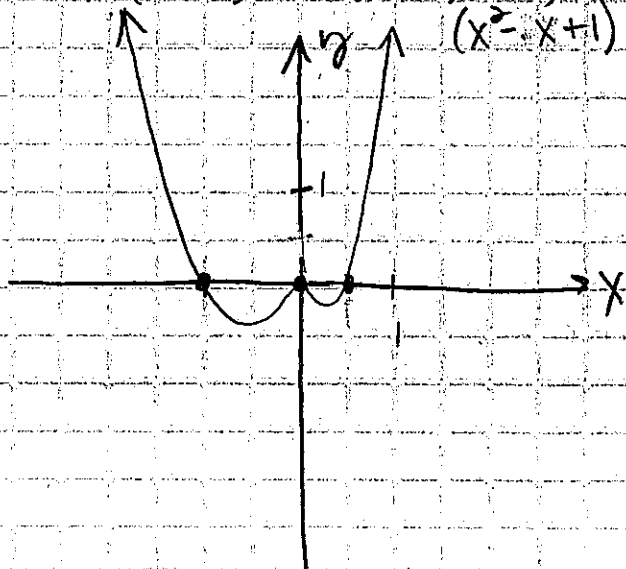
7.  $f(x) = -12x^2(x+2)(x-5)^3(2x+9)$

$D=7 \uparrow \downarrow$



8.  $f(x) = 16x^8 + 14x^5 - 2x^2$   
 $= 2x^2(8x^6 + 7x^3 - 1)$   
 $= 2x^2(8x^3 - 1)(x^3 + 1)$   
 $= 2x^2(2x-1)(4x^2+2x+1)(x+1)$   
 $(x^2-x+1)$

Deg = 8  $\uparrow \uparrow$





18. Let  $x$  represent time. The time increases by a constant amount of 1. The bacteria populations are the  $y$ -values.

First differences: 68 140 263 434

Second differences: 72 123 171

Third differences: 51 48

The third differences are relatively close, a cubic function should be a good model.

$$f(x) = 8.25x^3 - 13.18x^2 + 49.58x - 0.6$$

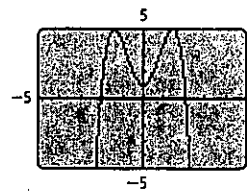
### STUDY GUIDE: REVIEW, PAGES 474-477

- monomial
- synthetic division
- multiplicity
- end behavior

### LESSON 6-1

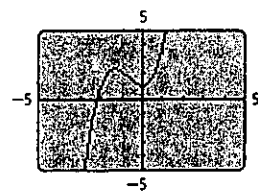
- Standard form:  $-3x^3 + 4x^2 + 6x + 7$   
Leading coefficient:  $-3$   
Degree: 3  
Terms: 4  
Name: cubic polynomial with 4 terms
- Standard form:  $-x^5 + 2x^4 + 5x^3 + 8x$   
Leading coefficient:  $-1$   
Degree: 5  
Terms: 4  
Name: quintic polynomial with 4 terms
- Standard form:  $9x^2 - 11x + 1$   
Leading coefficient: 9  
Degree: 2  
Terms: 3  
Name: quadratic trinomial
- Standard form:  $x^4 - 6x^2$   
Leading coefficient: 1  
Degree: 4  
Terms: 2  
Name: quartic binomial
- $(8x^3 - 4x^2 - 3x + 1) - (1 - 5x^2 + x)$   
 $= (8x^3 - 4x^2 - 3x + 1) + (5x^2 - x - 1)$   
 $= (8x^3) + (-4x^2 + 5x^2) + (-3x - x) + (1 - 1)$   
 $= 8x^3 + x^2 - 4x$
- $(6x^2 + 7x - 2) + (1 - 5x^3 + 3x)$   
 $= (6x^2 + 7x - 2) + (-5x^3 + 3x + 1)$   
 $= (-5x^3) + (6x^2) + (7x + 3x) + (-2 + 1)$   
 $= -5x^3 + 6x^2 + 10x - 1$
- $(5x - 2x^2) - (4x^2 + 6x - 9)$   
 $= (-2x^2 + 5x) + (-4x^2 - 6x + 9)$   
 $= (-2x^2 - 4x^2) + (5x - 6x) + 9$   
 $= -6x^2 - x + 9$
- $(x^4 - x^2 + 4) + (x^2 - x^3 - 5x^4 - 7)$   
 $= (x^4 - x^2 + 4) + (-5x^4 - x^3 + x^2 - 7)$   
 $= (x^4 - 5x^4) + (-x^3) + (-x^2 + x^2) + (4 - 7)$   
 $= -4x^4 - x^3 - 3$

13.



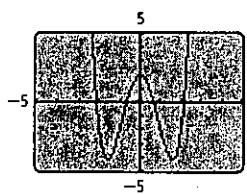
From left to right, it alternately increases and decreases, changing direction 3 times and crossing the  $x$ -axis 2 times. There appear to be 2 real zeros.

14.



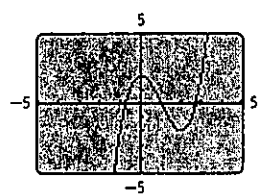
From left to right, it increases, decreases slightly, and then increases again. It crosses the  $x$ -axis 1 time. There appears to be 1 real zero.

15.



From left to right, it alternately decreases and increases, changing direction 3 times. It crosses the  $x$ -axis 4 times. There appear to be 4 real zeros.

16.



From left to right, it increases, decreases, and then increases again. It crosses the  $x$ -axis 3 times. There appear to be 3 real zeros.

### LESSON 6-2

- $5x^2(3x - 2)$   
 $= 5x^2(3x) + 5x^2(-2)$   
 $= 15x^3 - 10x^2$
- $-3t(2t^2 - 6t + 1)$   
 $= -3t(2t^2) - 3t(-6t) - 3t(1)$   
 $= -6t^3 + 18t^2 - 3t$
- $ab^2(a^2 - a + ab)$   
 $= ab^2(a^2) + ab^2(-a) + ab^2(ab)$   
 $= a^3b^2 - a^2b^2 + a^2b^3$
- $(x - 2)(x^2 - 2x - 3)$   
 $= x(x^2) + x(-2x) + x(-3) - 2(x^2) - 2(-2x) - 2(-3)$   
 $= x^3 - 2x^2 - 3x - 2x^2 + 4x + 6$   
 $= x^3 - 4x^2 + x + 6$
- $(2x + 5)(x^3 - x^2 + 1)$   
 $= 2x(x^3) + 2x(-x^2) + 2x(1) + 5(x^3) + 5(-x^2) + 5(1)$   
 $= 2x^4 - 2x^3 + 2x + 5x^3 - 5x^2 + 5$   
 $= 2x^4 + 3x^3 - 5x^2 + 2x + 5$
- $(x - 3)^3$   
 $= [1(x)^3(-3)^0] + [3(x)^2(-3)^1] + [3(x)^1(-3)^2]$   
 $+ [1(x)^0(-3)^3]$   
 $= x^3 - 9x^2 + 27x - 27$

$$\begin{aligned}
 23. & (x+4)(x^4 - 3x^2 + x) - \\
 & = x(x^4) + x(-3x^2) + x(x) + 4(x^4) + 4(-3x^2) + 4(x) \\
 & = x^5 - 3x^3 + x^2 + 4x^4 - 12x^2 + 4x \\
 & = x^5 + 4x^4 - 3x^3 - 11x^2 + 4x
 \end{aligned}$$

$$\begin{aligned}
 24. & (2x+1)^4 \\
 & = [1(2x)^4(1)^0] + [4(2x)^3(1)^1] + [6(2x)^2(1)^2] \\
 & \quad + [4(2x)^1(1)^3] + [1(2x)^0(1)^4] \\
 & = 16x^4 + 32x^3 + 24x^2 + 8x + 1
 \end{aligned}$$

$$\begin{aligned}
 25. & V = \pi r^2 \cdot h \\
 & = (\pi(2x)^2)(x^2 - x - 3) \\
 & = 4\pi x^2(x^2 - x - 3) \\
 & = 4\pi x^2(x^2) + 4\pi x^2(-x) + 4\pi x^2(-3) \\
 & = 4\pi x^4 - 4\pi x^3 - 12\pi x^2
 \end{aligned}$$

### LESSON 6-3

$$\begin{array}{r}
 26. \quad \begin{array}{r} x^2 - 7x + 16 \\ x+2 \overline{) x^3 - 5x^2 + 2x - 7} \\ \underline{-(x^3 + 2x^2)} \phantom{- 7} \\ -7x^2 + 2x \phantom{- 7} \\ \underline{-(-7x^2 - 14x)} \phantom{- 7} \\ 16x - 7 \\ \underline{-(16x + 32)} \\ -39 \end{array} \\
 \frac{x^3 - 5x^2 + 2x - 7}{x+2} = x^2 - 7x + 16 - \frac{39}{x+2}
 \end{array}$$

$$\begin{array}{r}
 27. \quad \begin{array}{r} 4x^3 + 2x^2 + 4x + 1 \\ 2x-1 \overline{) 8x^4 + 0x^3 + 6x^2 - 2x + 4} \\ \underline{-(8x^4 - 4x^3)} \phantom{+ 6x^2 - 2x + 4} \\ 4x^3 + 6x^2 \phantom{- 2x + 4} \\ \underline{-(4x^3 - 2x^2)} \phantom{- 2x + 4} \\ 8x^2 - 2x \phantom{- 2x + 4} \\ \underline{-(8x^2 - 4x)} \phantom{- 2x + 4} \\ 2x + 4 \\ \underline{-(2x - 1)} \\ 5 \end{array}
 \end{array}$$

$$\begin{aligned}
 & \frac{8x^4 + 6x^2 - 2x + 4}{2x-1} \\
 & = 4x^3 + 2x^2 + 4x + 1 + \frac{5}{2x-1}
 \end{aligned}$$

$$\begin{array}{r}
 28. \quad \begin{array}{r} 3 \phantom{0} 1 \phantom{0} -4 \phantom{0} 3 \phantom{0} 2 \\ \phantom{0} 3 \phantom{0} -3 \phantom{0} 0 \\ \hline 1 \phantom{0} -1 \phantom{0} 0 \phantom{0} 2 \end{array} \\
 \frac{x^3 - 4x^2 + 3x + 2}{x-3} = x^2 - x + \frac{2}{x-3}
 \end{array}$$

$$\begin{array}{r}
 29. \quad \begin{array}{r} 2 \phantom{0} 1 \phantom{0} 0 \phantom{0} 2 \phantom{0} -1 \\ \phantom{0} 2 \phantom{0} 4 \phantom{0} 12 \\ \hline 1 \phantom{0} 2 \phantom{0} 6 \phantom{0} 11 \end{array} \\
 \frac{x^3 + 2x - 1}{x-2} = x^2 + 2x + 6 + \frac{11}{x-2}
 \end{array}$$

$$\begin{aligned}
 30. \text{ Number of ribbons} & = \frac{\text{length of spool}}{\text{length of ribbon}} \\
 & = \frac{x^3 + x^2}{x-1}
 \end{aligned}$$

$$\begin{array}{r}
 \phantom{0} 1 \phantom{0} 1 \phantom{0} 0 \phantom{0} 0 \\
 \phantom{0} 1 \phantom{0} 2 \phantom{0} 2 \\
 \hline 1 \phantom{0} 2 \phantom{0} 2 \phantom{0} 2
 \end{array}$$

Yes. The number of strips of ribbons can be represented by  $x^2 + 2x + 2$ . Possible answer: with a remainder of 2 in.

### LESSON 6-4

$$\begin{array}{r}
 31. \quad \begin{array}{r} -3 \phantom{0} 1 \phantom{0} 2 \phantom{0} 0 \phantom{0} -5 \\ \phantom{0} -3 \phantom{0} 3 \phantom{0} -9 \\ \hline 1 \phantom{0} -1 \phantom{0} 3 \phantom{0} -14 \end{array} \\
 x+3 \text{ is not a factor of } P(x).
 \end{array}$$

$$\begin{array}{r}
 32. \quad \begin{array}{r} 1 \phantom{0} 4 \phantom{0} 0 \phantom{0} -5 \phantom{0} 3 \phantom{0} -2 \\ \phantom{0} 4 \phantom{0} 4 \phantom{0} -1 \phantom{0} 2 \\ \hline 4 \phantom{0} 4 \phantom{0} -1 \phantom{0} 2 \phantom{0} 0 \end{array} \\
 x-1 \text{ is a factor of } P(x).
 \end{array}$$

$$\begin{array}{r}
 33. \quad \begin{array}{r} 2 \phantom{0} 2 \phantom{0} -3 \phantom{0} 1 \phantom{0} -6 \\ \phantom{0} 4 \phantom{0} 2 \phantom{0} 6 \\ \hline 2 \phantom{0} 1 \phantom{0} 3 \phantom{0} 0 \end{array} \\
 x-2 \text{ is a factor of } P(x).
 \end{array}$$

$$\begin{aligned}
 34. & x^3 - x^2 - 16x + 16 \\
 & = (x^3 - x^2) + (-16x + 16) \\
 & = x^2(x-1) - 16(x-1) \\
 & = (x-1)(x^2 - 16) \\
 & = (x-1)(x-4)(x+4)
 \end{aligned}$$

$$\begin{aligned}
 35. & 4x^3 - 8x^2 - x + 2 \\
 & = (4x^3 - 8x^2) + (-x + 2) \\
 & = 4x^2(x-2) - (x-2) \\
 & = (x-2)(4x^2 - 1) \\
 & = (x-2)(2x-1)(2x+1)
 \end{aligned}$$

$$\begin{aligned}
 36. & 3x^3 + 81 \\
 & = 3(x^3 + 27) \\
 & = 3(x^3 + 3^3) \\
 & = 3(x+3)(x^2 - x \cdot 3 + 3^2) \\
 & = 3(x+3)(x^2 - 3x + 9)
 \end{aligned}$$

$$\begin{aligned}
 37. & 16x^3 - 2 \\
 & = 2(8x^3 - 1) \\
 & = 2[(2x)^3 - 1^3] \\
 & = 2(2x-1)[(2x)^2 + 2x \cdot 1 + 1^2] \\
 & = 2(2x-1)(4x^2 + 2x + 1)
 \end{aligned}$$

### LESSON 6-5

$$\begin{aligned}
 38. & x^3 - 5x^2 + 8x - 4 = 0 \\
 & \text{Possible rational roots: } \pm 1, \pm 5
 \end{aligned}$$

$$\begin{array}{r}
 \phantom{0} 1 \phantom{0} 1 \phantom{0} -5 \phantom{0} 8 \phantom{0} -4 \\
 \phantom{0} 1 \phantom{0} -4 \phantom{0} 4 \\
 \hline 1 \phantom{0} -4 \phantom{0} 4 \phantom{0} 0
 \end{array}$$

$$\begin{aligned}
 & (x-1)(x^2 - 4x + 4) = 0 \\
 & (x-1)(x-2)(x-2) = 0 \\
 & \text{The roots are 1 and 2.}
 \end{aligned}$$

39.  $x^3 + 6x^2 + 9x + 2 = 0$

Possible rational roots:  $\pm 1, \pm 2$

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 9 & 2 \\ & & -2 & -8 & -2 \\ \hline & 1 & 4 & 1 & 0 \end{array}$$

$(x + 2)(x^2 + 4x + 1) = 0$

Solve  $x^2 + 4x + 1 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4}}{2} = -2 \pm \sqrt{3}$

The fully factored equation is:

$(x + 2)[x - (-2 + \sqrt{3})][x - (-2 - \sqrt{3})] = 0$

The roots are  $-2$  and  $-2 \pm \sqrt{3}$ .

40.  $x^3 + 3x^2 + 3x + 1 = 0$

$(x + 1)^3 = 0$

The roots are  $-1$  with multiplicity 3.

41.  $x^4 - 12x^2 + 27 = 0$

$(x^2 - 9)(x^2 - 3) = 0$

$(x + 3)(x - 3)(x^2 - 3) = 0$

Solve  $x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

The fully factored equation is:

$(x + 3)(x - 3)(x + \sqrt{3})(x - \sqrt{3}) = 0$

The roots are  $-3, 3,$  and  $\pm\sqrt{3}$ .

42.  $x^3 + x^2 - 2x - 2 = 0$

$x^2(x + 1) - 2(x + 1) = 0$

$(x + 1)(x^2 - 2) = 0$

Solve  $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

The fully factored equation is:

$(x + 1)(x + \sqrt{2})(x - \sqrt{2}) = 0$

The roots are  $-1,$  and  $\pm\sqrt{2}$ .

43.  $x^3 - 5x^2 + 4 = 0$

Possible rational roots:  $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 0 & 4 \\ & & 1 & -4 & -4 \\ \hline & 1 & -4 & -4 & 0 \end{array}$$

$(x - 1)(x^2 - 4x - 4) = 0$

Solve  $x^2 - 4x - 4 = 0$

$x = \frac{4 \pm \sqrt{16 + 16}}{2} = 2 \pm 2\sqrt{2}$

The fully factored equation is:

$(x - 1)[x - (2 + 2\sqrt{2})][x - (2 - 2\sqrt{2})] = 0$

The roots are  $1,$  and  $2 \pm 2\sqrt{2}$ .

44. Let  $x$  represent the width in meters.

Then the length is  $2x,$  and the height is  $x + 4.$

$2x(x)(x + 4) = 48$

$(2x^2)(x + 4) = 48$

$2x^3 + 8x^2 = 48$

$2x^3 + 8x^2 - 48 = 0$

$2(x^3 + 4x^2 - 24) = 0$

Factors of  $-24:$   $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$\frac{p}{q}$	1	4	0	-24
1	1	5	5	-19
2	1	6	12	0
3	1	7	21	39

$2(x - 2)(x^2 + 6x + 12) = 0$

Solve  $x^2 + 6x + 12 = 0$

$x = \frac{-6 \pm \sqrt{36 - 48}}{2} = -3 \pm i\sqrt{3}$

The width must be positive and real, so the width should be  $2$  m.

**LESSON 6-6**

45.  $P(x) = (x + 3)(x - 2)(x - 4)$   
 $= (x^2 + x - 6)(x - 4)$   
 $= x^3 - 3x^2 - 10x + 24$

46.  $P(x) = (x + \frac{1}{2})(x + 2)(x - 3)$   
 $= (x^2 + \frac{5}{2}x + 1)(x - 3)$   
 $= x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3$

47.  $P(x) = (x - \sqrt{2})(x + \sqrt{2})(x + 1)$   
 $= (x^2 - 2)(x + 1)$   
 $= x^3 + x^2 - 2x - 2$

48.  $P(x) = (x + 3)(x - 1)(x + 1)$   
 $= (x + 3)(x^2 + 1)$   
 $= x^3 + 3x^2 + x + 3$

49.  $P(x) = (x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})$   
 $= (x^2 - 2)(x^2 - 3)$   
 $= x^4 - 5x^2 + 6$

50.  $P(x) = [x - (1 + \sqrt{3})][x - (1 - \sqrt{3})](x - 2)(x + 2)$   
 $= (x^2 - 2x - 2)(x^2 + 4)$   
 $= x^4 - 2x^3 + 2x^2 - 8x - 8$

51.  $x^3 - x^2 + 4x - 4 = 0$   
 $x^2(x - 1) + 4(x - 1) = 0$   
 $(x - 1)(x^2 + 4) = 0$   
Solve  $x^2 + 4 = 0$   
 $x^2 = -4$   
 $x = \pm 2i$

The fully factored equation is:

$(x - 1)(x - 2i)(x + 2i) = 0$

The solutions are  $1, 2i,$  and  $-2i.$

52.  $x^4 - x^2 - 2 = 0$   
 $(x^2 + 1)(x^2 - 2) = 0$   
 Solve  $x^2 + 1 = 0$        $x^2 - 2 = 0$   
 $x^2 = -1$        $x^2 = 2$   
 $x = \pm i$        $x = \pm\sqrt{2}$

The fully factored equation is:  
 $(x - i)(x + i)(x - \sqrt{2})(x + \sqrt{2}) = 0$   
 The solutions are  $i$ ,  $-i$ ,  $\sqrt{2}$ , and  $-\sqrt{2}$ .

53.  $x^4 - \frac{63}{4}x^2 - 4 = 0$   
 Possible rational roots:  $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -\frac{63}{4} & 0 & -4 \\ & & 4 & 16 & 1 & 4 \\ \hline & 1 & 4 & \frac{1}{4} & 1 & 0 \end{array}$$

$(x - 4)(x^3 + 4x^2 + \frac{1}{4}x + 1) = 0$   
 $(x - 4)(x + 4)(x^2 + \frac{1}{4}) = 0$

Solve  $x^2 + \frac{1}{4} = 0$   
 $x^2 = -\frac{1}{4}$   
 $x = \pm \frac{1}{2}i$

The fully factored equation is:  
 $(x - 4)(x + 4)(x - \frac{1}{2}i)(x + \frac{1}{2}i) = 0$   
 The solutions are  $4$ ,  $-4$ ,  $\frac{1}{2}i$ , and  $-\frac{1}{2}i$ .

54.  $x^3 + 3x^2 - 5x - 15 = 0$   
 $(x + 3)(x^2 - 5) = 0$   
 Solve  $x^2 - 5 = 0$   
 $x^2 = 5$   
 $x = \pm\sqrt{5}$

The fully factored equation is:  
 $(x + 3)(x - \sqrt{5})(x + \sqrt{5}) = 0$   
 The solutions are  $-3$ ,  $\sqrt{5}$ , and  $-\sqrt{5}$ .

**LESSON 6-7**

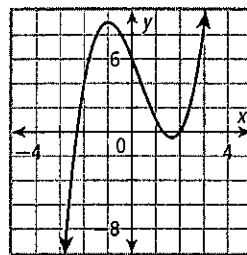
55. Leading coefficient:  $-2$ ; Degree:  $3$ ;  
 End behavior:  $x \rightarrow -\infty, f(x) \rightarrow +\infty$   
 $x \rightarrow +\infty, f(x) \rightarrow -\infty$ .
56. Leading coefficient:  $1$ ; Degree:  $4$ ;  
 End behavior:  $x \rightarrow -\infty, f(x) \rightarrow +\infty$   
 $x \rightarrow +\infty, f(x) \rightarrow +\infty$ .
57. Leading coefficient:  $-3$ ; Degree:  $6$ ;  
 End behavior:  $x \rightarrow -\infty, f(x) \rightarrow -\infty$   
 $x \rightarrow +\infty, f(x) \rightarrow -\infty$ .
58. Leading coefficient:  $7$ ; Degree:  $5$ ;  
 End behavior:  $x \rightarrow -\infty, f(x) \rightarrow -\infty$   
 $x \rightarrow +\infty, f(x) \rightarrow +\infty$ .

59.  $f(x) = x^3 - x^2 - 5x + 6$   
 Possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -5 & 6 \\ & & 2 & 2 & -6 \\ \hline & 1 & 1 & -3 & 0 \end{array}$$

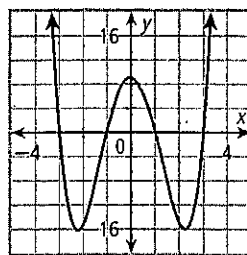
$f(x) = (x - 2)(x^2 + x - 3)$   
 Solve  $x^2 + x - 3 = 0$   
 $x = \frac{-1 \pm \sqrt{1 + 12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$

The zeros are  $2, \approx 1.303$ , and  $\approx -2.303$   
 Plot other points:  $f(0) = 6$ , so the  $y$ -intercept is  $6$ ;  
 $f(-1) = 9$ , and  $f(1.5) = -0.375$ .  
 The degree is odd and the leading coefficient is positive, so as  $x \rightarrow -\infty, P(x) \rightarrow -\infty$ ,  
 and as  $x \rightarrow +\infty, P(x) \rightarrow +\infty$ .



60.  $f(x) = x^4 - 10x^2 + 9$   
 $= (x^2 - 1)(x^2 - 9)$   
 $= (x - 1)(x + 1)(x - 3)(x + 3)$

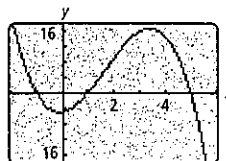
The zeros are  $1, -1, 3$ , and  $-3$ .  
 Plot other points:  $f(0) = 9$ , so the  $y$ -intercept is  $9$ ;  
 $f(-2) = -15$ , and  $f(2) = -15$ .  
 The degree is even and the leading coefficient is positive, so as  $x \rightarrow -\infty, P(x) \rightarrow +\infty$ ,  
 and as  $x \rightarrow +\infty, P(x) \rightarrow +\infty$ .



61.  $f(x) = -x^3 + 5x^2 + x - 5$   
 $= -(x^3 - 5x^2 - x + 5)$   
 $= -(x - 5)(x^2 - 1)$   
 $= -(x - 5)(x - 1)(x + 1)$

The zeros are  $5, 1$ , and  $-1$ .  
 Plot other points:  $f(0) = -5$ , so the  $y$ -intercept is  $-5$ ;  
 $f(3) = 16$

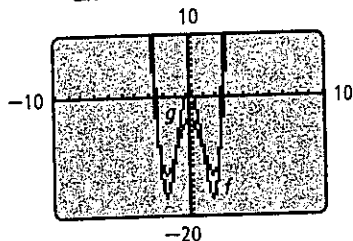
The degree is odd and the leading coefficient is negative, so as  $x \rightarrow -\infty, P(x) \rightarrow +\infty$ ,  
 and as  $x \rightarrow +\infty, P(x) \rightarrow -\infty$ .



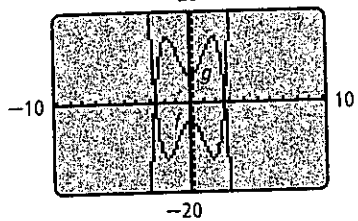


### LESSON 6-8

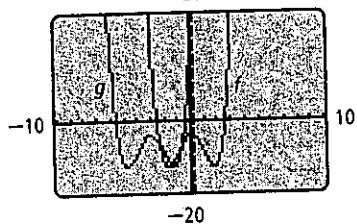
$$\begin{aligned}
 62. g(x) &= 2f(x) + 9 \\
 &= 2(x^4 - 6x^2 - 4) + 9 \\
 &= 2x^4 - 12x^2 - 8 + 9 \\
 &= 2x^4 - 12x^2 + 1
 \end{aligned}$$



$$\begin{aligned}
 63. g(x) &= -(f(x) - 2) \\
 &= -(x^4 - 6x^2 - 4 - 2) \\
 &= -(x^4 - 6x^2 - 6) \\
 &= -x^4 + 6x^2 + 6
 \end{aligned}$$



$$\begin{aligned}
 64. g(x) &= f(-x - 3) \\
 &= (-x - 3)^4 - 6(-x - 3)^2 - 4
 \end{aligned}$$



### LESSON 6-9

65. Let  $x$  represent the number of days. The days increase by a constant amount of 1. The attendances for the movie are the  $y$ -values.
- |                     |     |    |      |    |
|---------------------|-----|----|------|----|
| First differences:  | 50  | 20 | 70   | 40 |
| Second differences: | -30 | 50 | -30  |    |
| Third differences:  |     | 80 | -80  |    |
| Fourth differences: |     |    | -160 |    |
- The fourth differences are constant, a quartic will be the best model.

$$f(x) \approx -6\frac{2}{3}x^4 + 80x^3 - 328\frac{1}{3}x^2 + 572x - 72$$

66. Let  $x$  represent the number of years. The years increase by a constant amount of 1. The populations are the  $y$ -values.

First differences:	783	702	1104	1989
Second differences:		-81	402	885
Third differences:			483	483

The third differences are constant, a cubic function should be a good model.

$$f(x) \approx 80.5x^3 - 523.5x^2 + 1790x + 544$$

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$$\begin{aligned}
 1. (3x^2 - x + 1) + (x) \\
 &= (3x^2) + (-x + x) + (1) \\
 &= 3x^2 + 1
 \end{aligned}$$

$$\begin{aligned}
 2. (6x^3 - 3x + 2) - (7x^3 + 3x + 7) \\
 &= (6x^3 - 3x + 2) + (-7x^3 - 3x - 7) \\
 &= (6x^3 - 7x^3) + (-3x - 3x) + (2 - 7) \\
 &= -x^3 - 6x - 5
 \end{aligned}$$

$$\begin{aligned}
 3. (y^2 + 3y^2 + 2) + (y^4 + y^3 - y^2 + 5) \\
 &= (4y^2 + 2) + (y^4 + y^3 - y^2 + 5) \\
 &= (y^4) + (y^3) + (4y^2 - y^2) + (2 + 5) \\
 &= y^4 + y^3 + 3y^2 + 7
 \end{aligned}$$

$$\begin{aligned}
 4. (4x^4 + x^2) - (x^3 - x^2 - 1) \\
 &= (4x^4 + x^2) + (-x^3 + x^2 + 1) \\
 &= (4x^4) + (-x^3) + (x^2 + x^2) + (1) \\
 &= 4x^4 - x^3 + 2x^2 + 1
 \end{aligned}$$

$$\begin{aligned}
 5. C(15) &= \frac{1}{10}(15)^3 - (15)^2 + 25 = 137.50 \\
 \text{The cost of manufacturing 15 units is } &\$137.50.
 \end{aligned}$$

$$\begin{aligned}
 6. xy(2x^4y + x^2y^2 - 3xy^3) \\
 &= xy(2x^4y) + xy(x^2y^2) + xy(-3xy^3) \\
 &= 2x^5y^2 + x^3y^3 - 3x^2y^4
 \end{aligned}$$

$$\begin{aligned}
 7. (t + 3)(2t^2 - t + 3) \\
 &= t(2t^2) + t(-t) + t(3) + 3(2t^2) + 3(-t) + 3(3) \\
 &= 2t^3 - t^2 + 3t + 6t^2 - 3t + 9 \\
 &= 2t^3 + 5t^2 + 9
 \end{aligned}$$

$$\begin{aligned}
 8. (x + 5)^3 \\
 &= [1(x)^3(5)^0] + [3(x)^2(5)^1] + [3(x)^1(5)^2] + [1(x)^0(5)^3] \\
 &= x^3 + 15x^2 + 75x + 125
 \end{aligned}$$

$$\begin{aligned}
 9. (2y + 3)^4 \\
 &= [1(2y)^4(3)^0] + [4(2y)^3(3)^1] + [6(2y)^2(3)^2] \\
 &\quad + [4(2y)^1(3)^3] + [1(2y)^0(3)^4] \\
 &= 16y^4 + 96y^3 + 216y^2 + 216y + 81
 \end{aligned}$$

$$\begin{array}{r}
 10. \underline{2} \ 5 \ -6 \ -8 \\
 \phantom{2} \ 10 \ 8 \\
 \hline
 \phantom{2} \ 5 \ 4 \ \underline{0} \\
 5x^2 - 6x - 8 = 5x + 4 \\
 x - 2
 \end{array}$$

$$\begin{array}{r}
 11. \phantom{2x-1} \ x^2 - 3x + 3 \\
 2x-1 \overline{) 2x^3 - 7x^2 + 9x - 4} \\
 \underline{-(2x^3 - x^2)} \phantom{+ 9x - 4} \\
 \phantom{2x-1} \ -6x^2 + 9x \phantom{- 4} \\
 \underline{-(-6x^2 + 3x)} \phantom{- 4} \\
 \phantom{2x-1} \phantom{-6x^2} \ 6x - 4 \phantom{- 4} \\
 \phantom{2x-1} \phantom{-6x^2} \ \underline{-(6x - 3)} \\
 \phantom{2x-1} \phantom{-6x^2} \phantom{6x} \ -1 \phantom{- 4}
 \end{array}$$

$$\frac{2x^3 - 7x^2 + 9x - 4}{2x - 1} = x^2 - 3x + 3 - \frac{1}{2x - 1}$$

$$\begin{array}{r}
 12. \underline{3} \ 1 \ 3 \ -1 \ 2 \ -6 \\
 \phantom{3} \ 3 \ 18 \ 51 \ 159 \\
 \hline
 \phantom{3} \ 1 \ 6 \ 17 \ 53 \ \underline{153}
 \end{array}$$

