Honors Geometry
Notes - Section 1.6 Midpoint and Distance in the Coordinate Plane

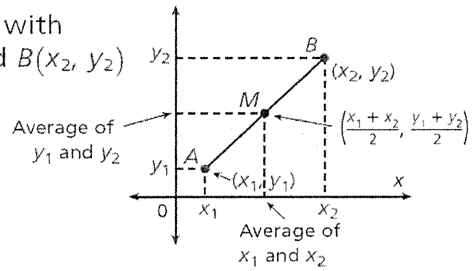
Name KEY
Date ____________ Period ________

- A **coordinate plane** is a plane that is divided into four regions by a horizontal line (x-axis) and a vertical line (y-axis). The location, or coordinates, of a point are given by an ordered pair (x, y).

Midpoint Formula

The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is found by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



Example 1: Find the coordinates of the midpoint of $\overline{PQ}$ with endpoints $P(-8, 3)$ and $Q(-2, 7)$.

$x_1 \; y_1 \quad x_2 \; y_2$

$$M = \left(\frac{-8+(-2)}{2}, \frac{3+7}{2}\right) = \left(\frac{-10}{2}, \frac{10}{2}\right) = (-5, 5)$$

Example 2: M is the midpoint of $\overline{XY}$. X has coordinates (2, 7) and M has coordinates (6, 1). Find the coordinates of Y. (2 METHODS)

(2,7) (6,1) (10,-5)

X M Y

$$(6, 1) = \left(\frac{x+2}{2}, \frac{y+7}{2}\right)$$

$6 = \frac{x+2}{2}$ $\qquad$ $1 = \frac{y+7}{2}$

$12 = x+2$ $\qquad$ $2 = y+7$

$x = 10$ $\qquad$ $y = -5$

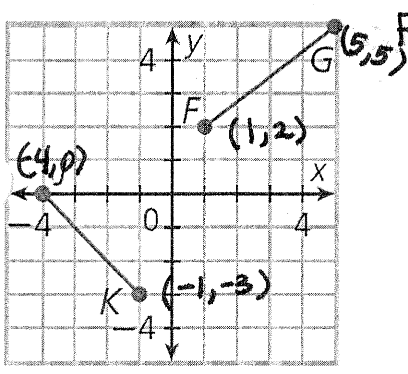
$(10, -5)$

Distance Formula

In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example 3: Find FG and JK. Then determine whether $\overline{FG} \cong \overline{JK}$.



$$FG = \sqrt{(5-1)^2 + (5-2)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$JK = \sqrt{(-4-(-1))^2 + (0-(-3))^2} = \sqrt{9+9} = \sqrt{18} \approx 4.24$$

$$\overline{FG} \not\cong \overline{JK}$$

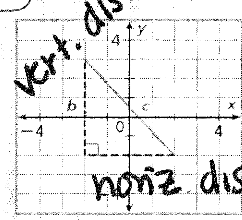
- You can also use the Pythagorean Theorem to find the distance between two points in a coordinate plane.

- In a right triangle, the two sides that form the right angle are the <u>legs</u>. The side across from the right angle that stretches from one leg to the other is the <u>hypotenuse</u>. In the diagram, *a* and *b* are the lengths of the shorter sides, or legs, of the right triangle. The longest side is called the hypotenuse and has length *c*.

Theorem 1-6-1 Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the *legs* is equal to the square of the length of the *hypotenuse*.

$$a^2 + b^2 = c^2$$



<u>Example 4:</u> Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from *D*(3, 4) to *E*(-2, -5).

ist. ormula

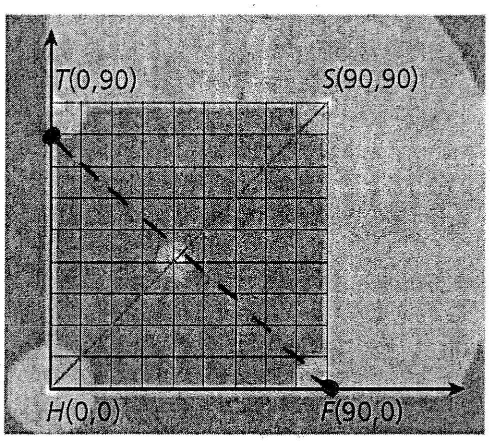
$$d = \sqrt{(3-(-2))^2 + (4-(-5))^2}$$
$$d = \sqrt{5^2 + 9^2}$$
$$d = \sqrt{25 + 81}$$
$$d = \sqrt{106} \approx 10.3$$

Pythag. Thm

horiz. vert.

$$x^2 + y^2 = d^2$$
$$5^2 + 9^2 = d^2$$
$$25 + 81 = d^2$$
$$106 = d^2$$
$$d \approx 10.3$$

<u>Example 5:</u> A player throws the ball from first base to a point located between third base and home plate and 10 feet from third base. What is the distance of the throw, to the nearest tenth?



$$d = \sqrt{(90-0)^2 + (0-80)^2}$$
$$d = \sqrt{90^2 + (-80)^2}$$
$$d = \sqrt{8100 + 6400}$$
$$d = \sqrt{14{,}500}$$
$$d \approx 120.4 \text{ feet}$$