

When you combine a conditional statement and its converse, you create a *biconditional statement*.

A **biconditional statement** is a statement that can be written in the form "*p* if and only if *q*."
This means "if *p*, then *q*" and "if *q*, then *p*."

$$p \leftrightarrow q \text{ means } p \rightarrow q \text{ and } q \rightarrow p$$

The biconditional "*p* if and only if *q*" can also be written as "*p* iff *q*" or $p \leftrightarrow q$.

Example 1: Write the conditional statement and converse within the biconditional.

- a. An angle is obtuse ^P if and only if its measure is greater than 90° and less than 180° . ^Q

Conditional: If an angle is obtuse, then the angle's measure is greater than 90° ; but less than 180° .
 $P \rightarrow Q$

Converse: If an angle's measure is greater than 90° ; but less than 180° ; then the angle is obtuse.
 $Q \rightarrow P$

- b. A solution is neutral ^P \leftrightarrow its pH is 7. ^Q

Conditional: If a solution is neutral, then its pH is 7.
 $P \rightarrow Q$

Converse: If a solution's pH is 7, then the solution is neutral.
 $Q \rightarrow P$

Example 2: For each conditional, write the converse and a biconditional statement.

- a. If $5x - 8 = 37$, then $x = 9$.

Converse: If $x = 9$, then $5x - 8 = 37$.

Biconditional: $5x - 8 = 37$ iff $x = 9$.

- b. If two angles have the same measure, then they are congruent.

Conditional: If two angles are congruent, then the angles have the same measure.

Biconditional: Two angles have the same measure iff they are congruent.

For a biconditional statement to be true both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.

Example 3: Determine if the biconditional is true. If false, give a counterexample.

a. A rectangle has side lengths of 12 cm and 25 cm ^P if and only if ^Q its area is 300 cm². ^F

T Conditional: If a rectangle has side lengths of 12 cm & 25 cm, then the area is 300 cm².

F Converse: If the area of a rectangle is 300 cm², then the rectangle has side lengths of 12 cm & 25 cm.

b. A natural number n is odd $\leftrightarrow n^2$ is odd. T

T Conditional: If a natural number ⁿ is odd, then n^2 is odd.

T Converse: If n^2 is odd, then n is an odd natural number.

c. An angle is a right angle ^P iff ^Q its measure is 90°. T

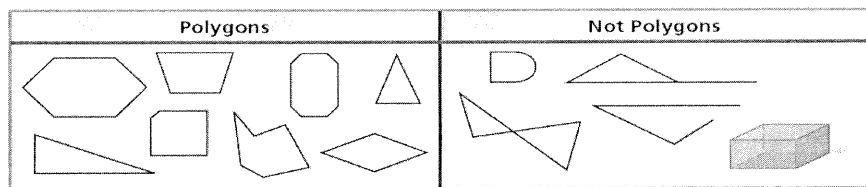
T Conditional: If an angle is a right angle, then its measure is 90°.

T Converse: If an angle's measure is 90°, then it is a right angle.

In geometry, biconditional statements are used to write *definitions*.

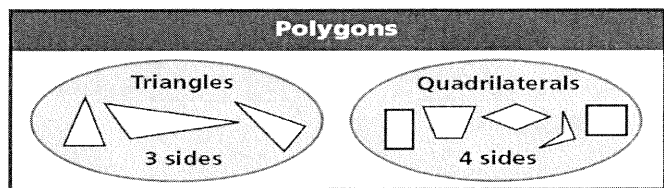
A **definition** is a statement that describes a mathematical object and can be written as a true biconditional.

- In the glossary, a **polygon** is defined as a closed plane figure formed by three or more line segments.



A figure is a polygon iff it is a closed plane figure formed by 3+ line segments

- A **triangle** is defined as a three-sided polygon, and a **quadrilateral** is a four-sided polygon.



A figure is a triangle iff it is a three-sided polygon.

A figure is a quadrilateral iff it is a four-sided polygon.

- Think of definitions as being reversible. Postulates, however are not necessarily true when reversed.