

# Triangles and Quadrilaterals in Standard Position

Standard Position of any triangle or quadrilateral:

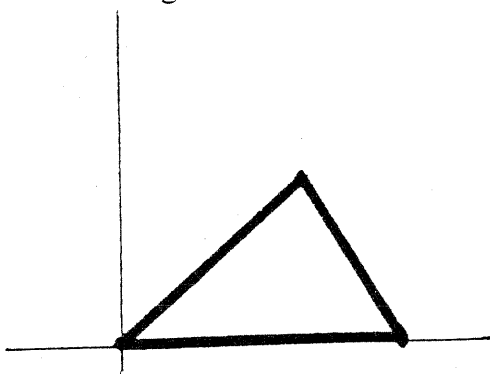
What do you notice about standard position?

⇒ Figure is positioned w/ a vertex at the origin  $(0,0)$ .

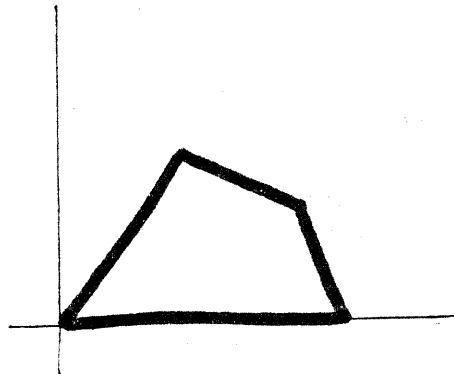
⇒ 1<sup>st</sup> Quad

⇒ Use  $x$ ,  $y$ -axes as boundaries!

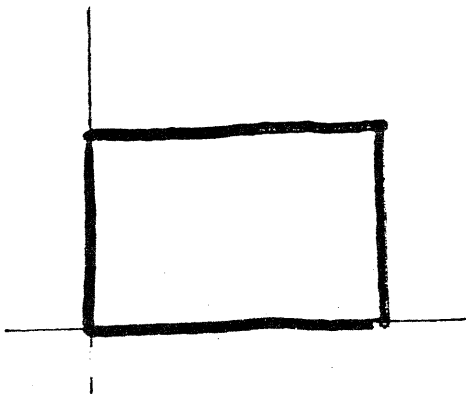
Triangle



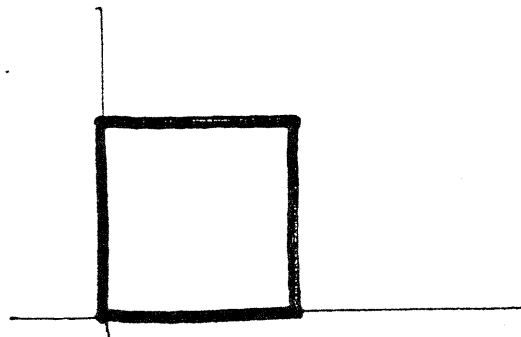
Quadrilateral



Rectangle



Square



**Coordinate Proof**

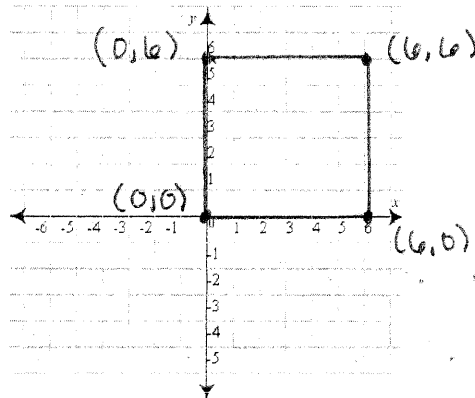
A style of proof that uses coordinate geometry and algebra. The first step is to position the given figure in the plane.

**Strategies for Positioning Figures in the Coordinate Plane**

*Use standard position if you can!*

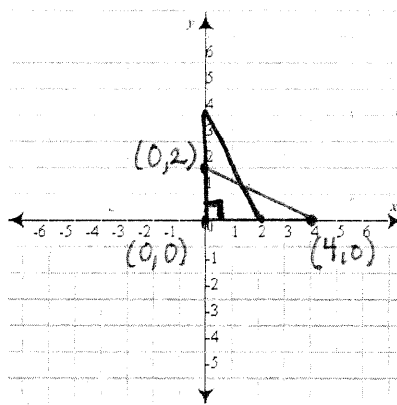
- ★ Use the origin as a vertex, keeping the figure in Quadrant I.
  - Center the figure at the origin.
  - Center a side of the figure at the origin.
- ☺ Use one or both axes as sides of the figure.

Example 1: Position a square with side length of 6 units in the coordinate plane. *(standard form)*



Example 2: Position a right triangle with leg lengths of 2 and 4 units in the coordinate plane.

*(standard form)*



NOTE: You can use slope, the coordinates of the vertices, the Distance Formula, or the Midpoint Formula to prove statements about the figure.

Slope Formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Midpoint Formula:

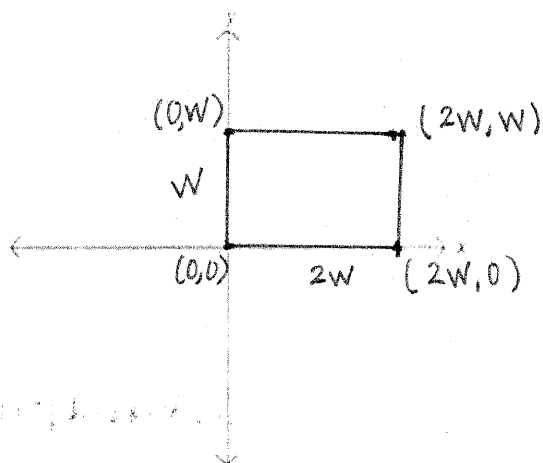
$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Distance Formula:

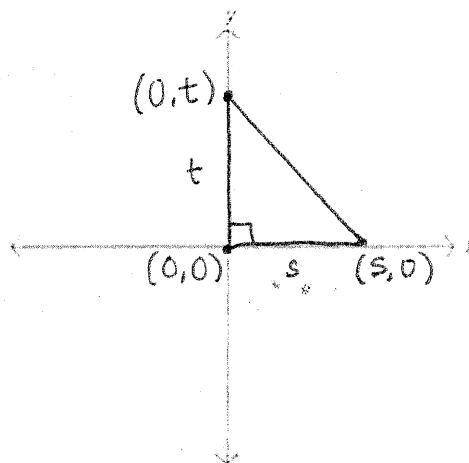
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 3: Position each figure in the coordinate plane and give the coordinates of each vertex.

A. rectangle with width  $w$  and length twice the width  $2w$



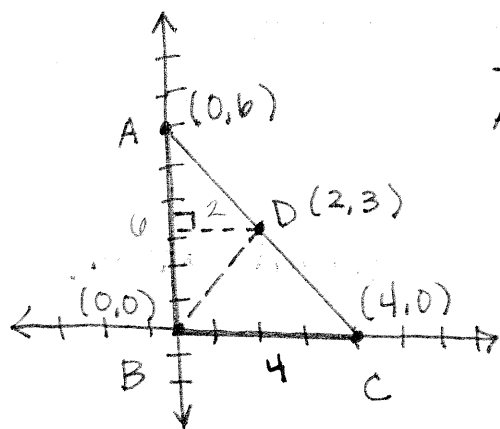
B. right triangle with legs of length  $s$  and  $t$ .



Example 4:

Given: Right  $\triangle ABC$  has vertices  $A(0,6)$ ,  $B(0,0)$ , and  $C(4,0)$ .  $D$  is the midpoint of  $AC$ .

Prove: Area of  $\triangle ADB$  is one half the area of  $\triangle ABC$ .



$$\text{Area } \triangle ABC$$

$$A = \frac{1}{2}(4)(6) = \frac{1}{2}(24) = \boxed{12 \text{ un}^2}$$

$$\text{Midpoint } D$$

$$M = \left( \frac{0+4}{2}, \frac{6+0}{2} \right) = (2, 3)$$

$$\text{Area } \triangle ADB$$

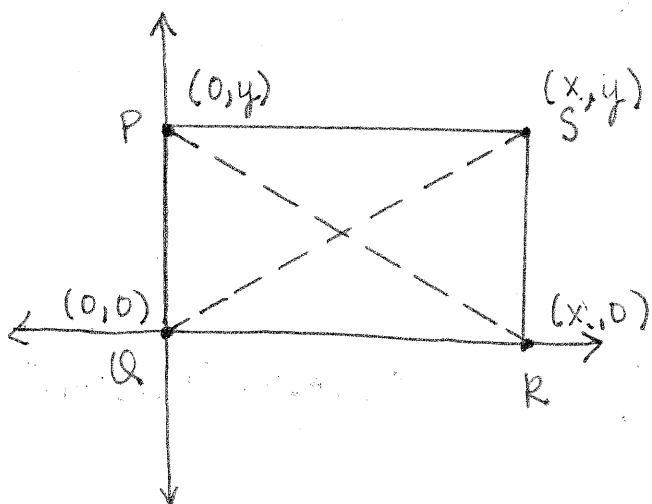
$$A = \frac{1}{2}(2)(6) = \frac{1}{2}(12) = \boxed{6 \text{ un}^2}$$

$\therefore$  Area of  $\triangle ADB$  is  $\frac{1}{2}(\triangle ABC)$ .

Example 5:

Given: Rectangle PQRS

Prove: Diagonals are congruent.



$$SQ = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

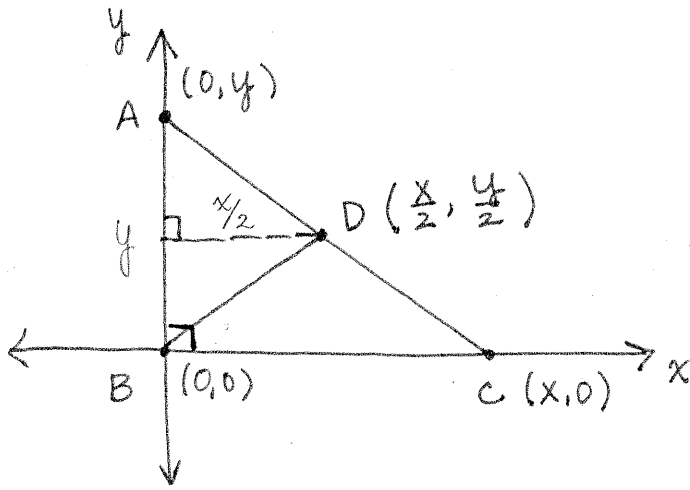
$$PR = \sqrt{(0-x)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

$SQ = PR$   
 $\therefore \overline{SQ} \cong \overline{PR}$

Example 6:

Given:  $\angle B$  is a right angle in  $\triangle ABC$ .  $D$  is the midpoint of  $\overline{AC}$ .

Prove: The area of  $\triangle ADB$  is one half the area of  $\triangle ABC$ .



$$\begin{aligned} &\text{Area } \triangle ABC \\ \hline A &= \frac{1}{2}(x)(y) = \boxed{\frac{xy}{2}} \end{aligned}$$

Midpoint D

$$M = \left( \frac{0+x}{2}, \frac{y+0}{2} \right) = \left( \frac{x}{2}, \frac{y}{2} \right)$$

Area  $\triangle ADB$

$$A = \frac{1}{2}(y)\left(\frac{x}{2}\right)$$

$$A = \frac{1}{2}\left(\frac{xy}{2}\right) = \boxed{\frac{xy}{4}}$$

Area of  
 $\therefore \triangle ADB$   
is  $\frac{1}{2} \triangle ABC$ .