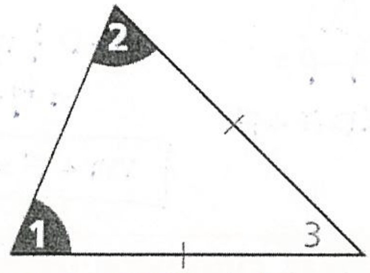
Honors Geometry
Notes - Section 4.8
Isosceles and Equilateral Triangles

Name KEY
Date ______________ Period ______

I. Isosceles Triangle



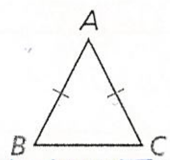
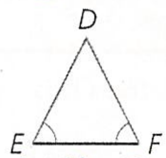
Legs: congruent sides of the Δ

Vertex Angle: Angle opposite the base

Base: Side opposite the vertex

Base angles: Angles opposite the congruent sides (legs)

Theorems Isosceles Triangle

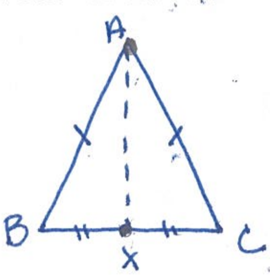
THEOREM		HYPOTHESIS	CONCLUSION
4-8-1	**Isosceles Triangle Theorem** If two sides of a triangle are congruent, then the angles opposite the sides are congruent.	 $\overline{AB} \cong \overline{AC}$	⇒ $\angle B \cong \angle C$
4-8-2	**Converse of Isosceles Triangle Theorem** If two angles of a triangle are congruent, then the sides opposite those angles are congruent.	 $\angle E \cong \angle F$	⇒ $\overline{DE} \cong \overline{DF}$

(skip)

Example 1: Proving the Isosceles Triangle Theorem: (p. 273)

Just use HL (informal!)

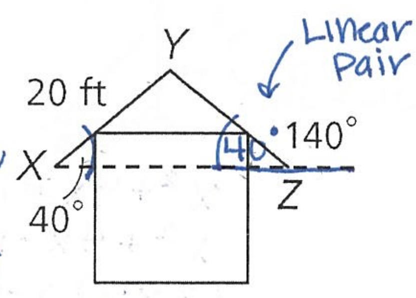
Given: $\overline{AB} \cong \overline{AC}$
Prove: $\angle B \cong \angle C$



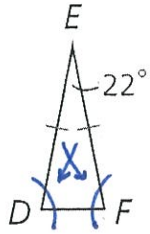
S	J
1. Draw in X, the midpt of $\overline{BC}$.	1. Every seg. has a unique midpt.
2. Draw in the aux. line $\overline{AX}$.	2. Through 2 pts, there is one line.
3. $\overline{AB} \cong \overline{AC}$	3. Given
4. $\overline{BX} \cong \overline{XC}$	4. Def. of midpt
5. $\overline{AX} \cong \overline{AX}$	5. Reflexive prop of =
6. $\triangle ABX \cong \triangle ACX$	6. SSS
7. $\angle B \cong \angle C$	7. CPCTC.

Example 2: The length of YX is 20 feet. Explain why the length of YZ is the same.

Because of the converse of the Isos. Δ Thm: The base angles both measure 40°, so the sides opp ⇒ $\overline{YX}$ and $\overline{YZ}$ must be ≅



Example 3: a. Find m∠F.



$\angle D \cong \angle F$

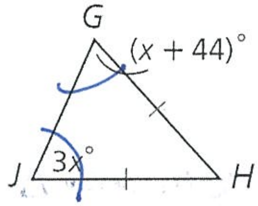
$180-22=\frac{158°}{2}$

$m\angle D \div m\angle F$

$=79°$

$\boxed{m\angle F=79°}$

b. Find m∠G.



$x+44=3x$

$44=2x$

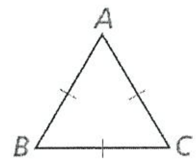
$x=22$

$m\angle G=22+44$

$\boxed{m\angle G=66°}$

II. Equilateral Triangle

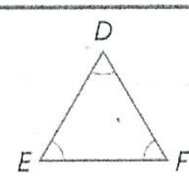
Corollary 4-8-3 Equilateral Triangle

COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equilateral, then it is equiangular. (equilateral △ → equiangular △)		∠A ≅ ∠B ≅ ∠C

go hand-in-hand

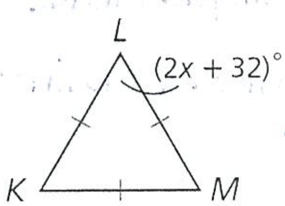
III. Equiangular Triangle

Corollary 4-8-4 Equiangular Triangle

COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equiangular, then it is equilateral. (equiangular △ → equilateral △)		$\overline{DE} \cong \overline{DF} \cong \overline{EF}$

Example 4: Find each value.

a. Find x.

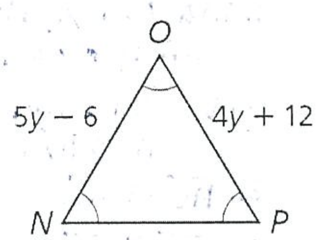


$2x+32=60$

$2x=28$

$\boxed{x=14}$

b. Find y.



$5y-6=4y+12$

$-6=-y+12$

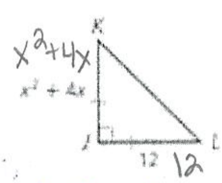
$-18=-y$

$\boxed{y=18}$

c. Find x.

Given: $\overline{JK} \cong \overline{JL}$

$\overline{JK} \cong \overline{JL}$



$x^2+4x=12$

$x^2+4x-12=0$

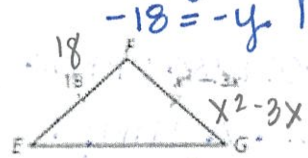
$(x+6)(x-2)=0$

$\boxed{\begin{array}{l}x=-6\\x=2\end{array}}$

d. Find x

Given: $\overline{FE} \cong \overline{FG}$

$\overline{FE} \cong \overline{FG}$



$18=x^2-3x$

$0=x^2-3x-18$

$0=(x-6)(x+3)$

$\boxed{\begin{array}{l}x=6\\x=-3\end{array}}$

IV. Using Coordinate Proofs with Isosceles and Equilateral Triangles

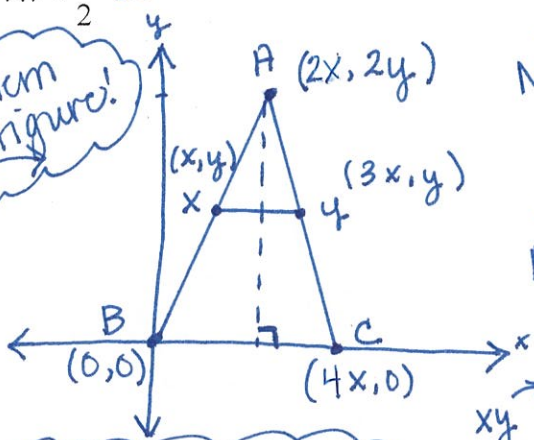
Example 5: Prove that the segment joining the midpoints of two sides of an isosceles triangle is half the base.

USE: $A(2x, 2y)$ $B(0,0)$ $C(4x, 0)$

Given: In isosceles $\triangle ABC$, X is the midpoint of $\overline{AB}$, and Y is the midpoint of $\overline{AC}$

Prove: $XY = \frac{1}{2}$ BC

Give them the figure!



*pick good pts.!

Midpoint of $\overline{AB}$

$$M = \left(\frac{2x+0}{2}, \frac{2y+0}{2}\right) = (x, y)$$

Midpoint of $\overline{AC}$

$$M = \left(\frac{2x+4x}{2}, \frac{2y+0}{2}\right) = (3x, y)$$

$$XY \rightarrow d = \sqrt{(x-3x)^2 + (y-y)^2} = \sqrt{(-2x)^2 + 0^2} = \sqrt{4x^2} = \boxed{2x}$$

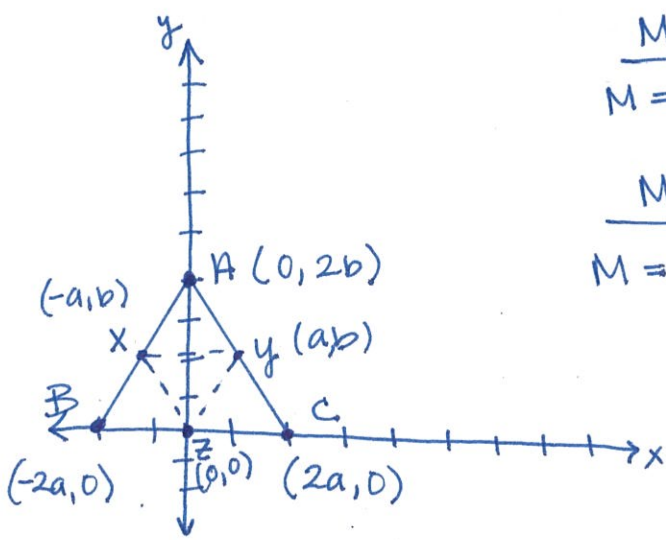
$$BC \rightarrow d = \sqrt{(0-4x)^2 + (0-0)^2} = \sqrt{(-4x)^2 + 0^2} = \sqrt{16x^2} = \boxed{4x}$$

$\therefore XY = \frac{1}{2} BC$

Example 6: What if...?

Given: The coordinates of isosceles $\triangle ABC$ are $A(0, 2b)$, $B(-2a, 0)$, and $C(2a, 0)$.
X is the midpoint of $\overline{AB}$, and Y is the midpoint of $\overline{AC}$, Z has coordinates (0,0).

Prove: $\triangle XYZ$ is isosceles.



Midpt of AB (X)

$$M = \left(\frac{0+-2a}{2}, \frac{2b+0}{2}\right) = (-a, b)$$

Midpt of AC (Y)

$$M = \left(\frac{0+2a}{2}, \frac{2b+0}{2}\right) = (a, b)$$

$$XZ = \sqrt{(-a-0)^2 + (b-0)^2} = \boxed{\sqrt{a^2+b^2}}$$

$$YZ = \sqrt{(a-0)^2 + (b-0)^2} = \boxed{\sqrt{a^2+b^2}}$$

must show at least 2 sides are ≅

$$XY = \sqrt{(-a-a)^2 + (\cancel{b-b})^2} = \sqrt{(-2a)^2} = \sqrt{4a^2} = \boxed{2a}$$

$XZ = YZ$

so, $\overline{XZ} \cong \overline{YZ}$

$\therefore \triangle XYZ$ is isosceles!