Honors Geometry
Notes -Section 5.3 Medians and Altitudes of Triangles

Name KEY
Date________________Period_______

HW: p. 318, #12-15, 17, 19, 21-26, 29-32

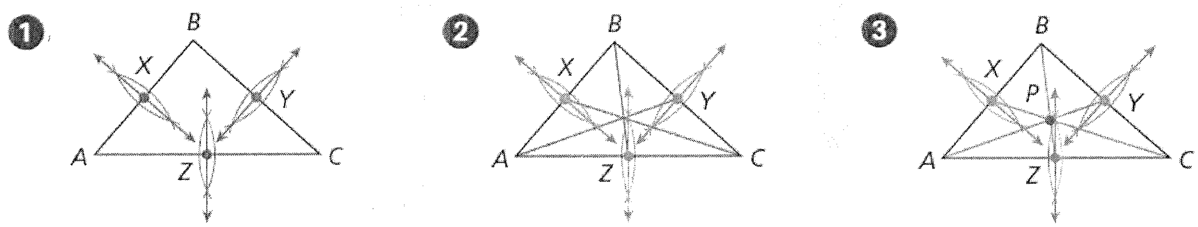
I. What is a median of a triangle?

A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side. Every triangle has three medians and the medians are concurrent.
(Doesn't have to be the ⊥ bisector)

The point of concurrency of the medians of a triangle is the **Centroid of the Triangle.**

- The centroid is always inside the triangle.

Construction Centroid of a Triangle

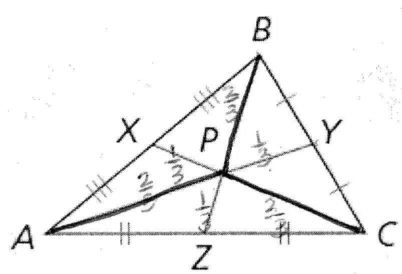


1. Draw $\triangle ABC$. Construct the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. Label the midpoints of the sides X, Y, and Z, respectively.
2. Draw $\overline{AY}$, $\overline{BZ}$, and $\overline{CX}$. These are the three medians of $\triangle ABC$.
3. Label the point where $\overline{AY}$, $\overline{BZ}$, and $\overline{CX}$ intersect as P.

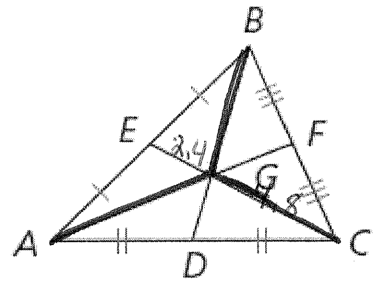
Theorem 5-3-1 Centroid Theorem

The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

$AP = \frac{2}{3}AY \qquad BP = \frac{2}{3}BZ \qquad CP = \frac{2}{3}CX$



<u>Example #1:</u> In ΔABC, $AF = 9$ and $GE = 2.4$. Find each length.



a) AG $= \frac{2}{3}(9) = 6$

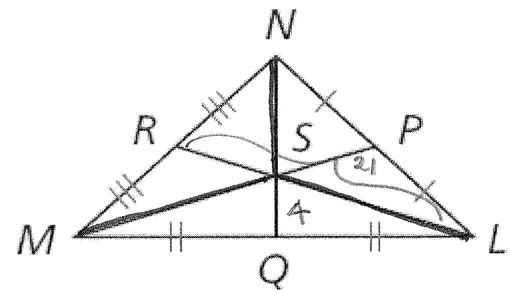
b) CE $= 7.2$

$\frac{1}{3}(EC) = 2.4$

$EC = 3(2.4)$

$EC = 7.2$

<u>Example #2:</u> In ΔLMN, $RL = 21$ and $SQ = 4$. Find each length.



a) LS $\frac{2}{3}(21) = \frac{42}{3} = 14$

b) NQ $\frac{1}{3}NQ = 4$

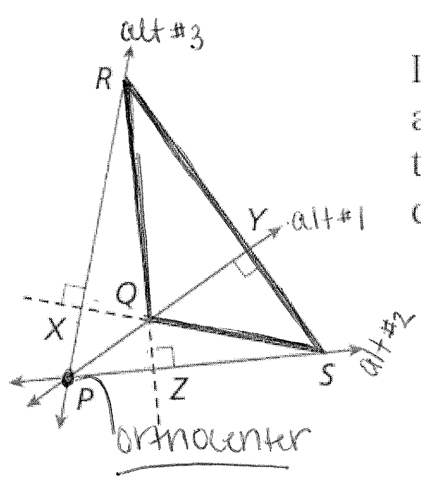
$NQ = 12$

II. What is an altitude of a triangle?

BUT NOT a BISECTOR!

An **<u>Altitude of a triangle</u>** is a <u>perpendicular segment from a vertex to the line containing the opposite side.</u> Every triangle has three altitudes and the altitudes are concurrent. An altitude can be inside, outside, or on the triangle.

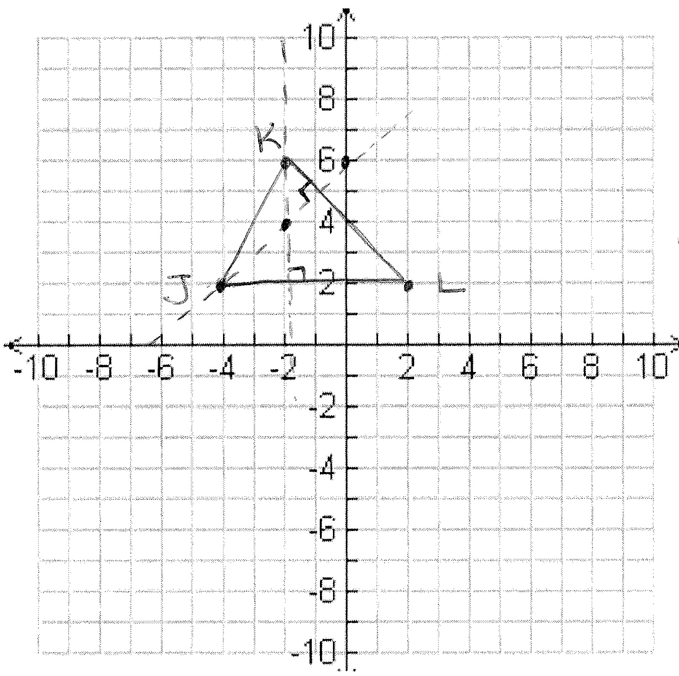
The point of concurrency of the altitudes of a triangle is the **<u>Orthocenter of the Triangle</u>**.



In $\triangle QRS$, altitude $\overline{QY}$ is inside the triangle, but $\overline{RX}$ and $\overline{SZ}$ are not. Notice that the lines containing the altitudes are concurrent at P. This point of concurrency is the <u>orthocenter</u> of the triangle.

Example #3: Find the orthocenter of ΔJKL with vertices $J(-4, 2)$, $K(-2, 6)$, $L(2, 2)$

* must find 2 of the 3 altitudes *



① $x = -2$

$m(KL) = \frac{2-6}{2-(-2)} = \frac{-4}{4} = -1$

$m(\perp bis) = 1$

$m = 1$, $(-4, 2)$ > $y = mx + b$

$2 = (1)(-4) + b$

$2 = -4 + b$

$b = 6$

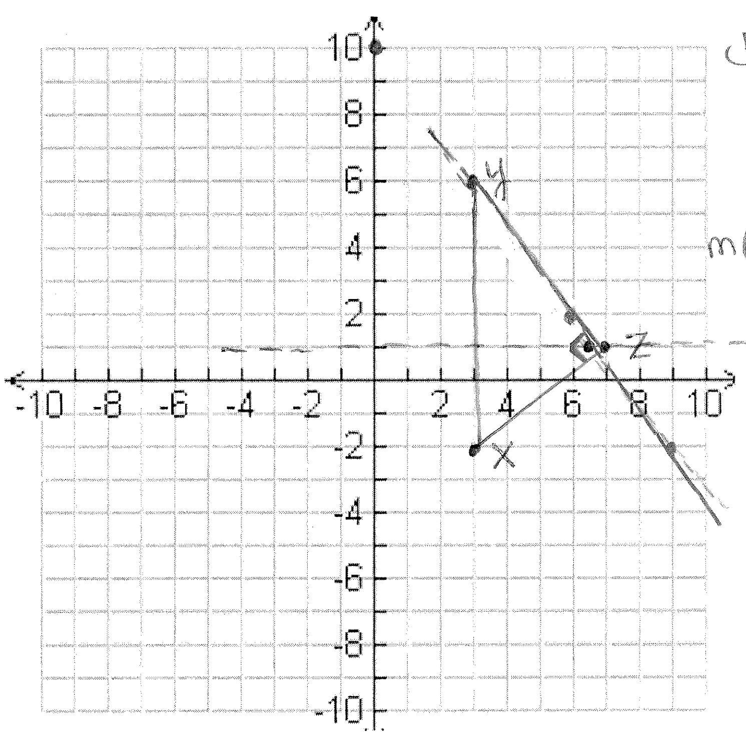
② $y = x + 6$

$\begin{cases} x = -2 \\ y = x + 6 \end{cases}$ $\quad y = -2 + 6$

$y = 4$

$\boxed{(-2, 4)}$

Example #4: Find the orthocenter of ΔXYZ with vertices $X(3, -2)$, $Y(3, 6)$, $Z(7, 1)$



① $y = 1$

$(3, 6)$

$m(XZ) = \frac{-2-1}{3-7} = \frac{-3}{-4} = \frac{3}{4}$

$m(\perp bis) = -\frac{4}{3}$

$6 = -\frac{4}{\cancel{3}}(\cancel{3}) + b$

$6 = -4 + b$

$b = 10$

② $y = -\frac{4}{3}x + 10$

$\begin{cases} y = 1 \\ y = -\frac{4}{3}x + 10 \end{cases} \Rightarrow 1 = -\frac{4}{3}x + 10$

$-9 = -\frac{4}{3}x$

$\frac{-27}{-4} = \frac{-4x}{-4}$ $\quad x = 6.75$

$\boxed{(6.75, 1)}$