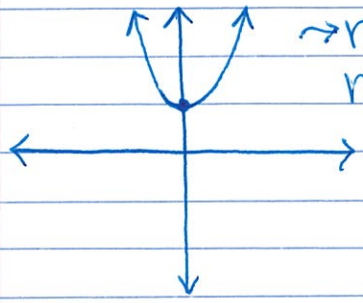
Section 5.5 - Complex Numbers

$f(x) = x^2 + 1 \rightsquigarrow 0 = x^2 + 1 \rightsquigarrow x = \pm\sqrt{-1}$

$-1 = x^2$

can't do this!



→ no real roots
never crosses the x axis

imaginary number – square root of a negative number

$\sqrt{-1} = i$

Simplify:

ex: $\sqrt{-24}$

$\sqrt{-1} \cdot \sqrt{24}$

$i \cdot \sqrt{4} \cdot \sqrt{6}$

$2i\sqrt{6}$

ex: $\sqrt{-72}$

$\sqrt{-1} \cdot \sqrt{72}$

$i \cdot \sqrt{36} \cdot \sqrt{2}$

$6i\sqrt{2}$

ex: $2\sqrt{-18}$

$2 \cdot \sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{2}$

$6i\sqrt{2}$

ex: $-5\sqrt{-125}$

$-5 \cdot \sqrt{-1} \cdot \sqrt{25} \cdot \sqrt{5}$

$-25i\sqrt{5}$

Solve: ex: $x^2 + 81 = 0$

$x^2 = -81$

$x = \pm 9i$

ex: $x^2 + 3x + 5 = 0$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(5)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{9 - 20}}{2}$

$x = \frac{-3 \pm \sqrt{-11}}{2}$

$x = \frac{-3 \pm i\sqrt{11}}{2}$

ex: $x^2 + 2x + 7 = 0$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4 - 28}}{2}$

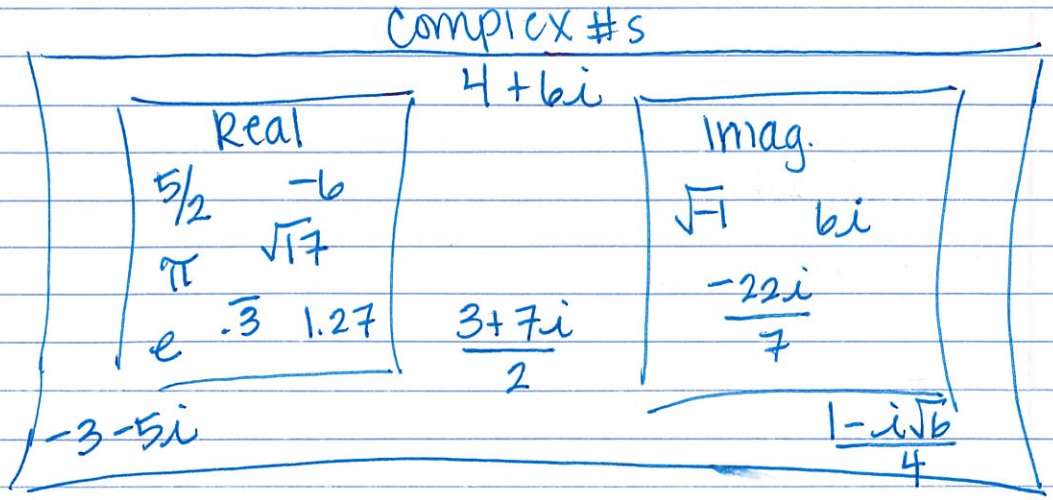
$x = \frac{-2 \pm \sqrt{-24}}{2}$

$x = \frac{-2 \pm 2i\sqrt{6}}{2}$

$x = -1 \pm i\sqrt{6}$

Complex Numbers:

- Every number is a complex number
- All numbers have a real & imag. part



Complex # → $a + bi$

a ↑ real part; bi ↑ imag. part

ex: $2+6i$ (R, I)

$5 - i\sqrt{7}$ (R, I)

$4 \rightarrow 4 + 0i$ (R, I)

$6i \rightarrow 0 + 6i$ (R, I)

2 complex numbers are equivalent iff the real parts are equal and the imag. parts are equal.

ex: $x + yi = 6 - 5i$
(R: x, 6; I: yi, $-5i$)

$x = 6, \quad y = -5$

ex: $(x-3) + (y+2)i = 4x - 3i$
(R: $x-3$, $4x$; I: $(y+2)i$, $-3i$)

$x - 3 = 4x$ $\qquad$ $y + 2 = -3$
$-3 = 3x$ $\qquad$ $y = -5$
$x = -1$

Complex roots (xint, zeros, sol.) come in conjugate pairs

→ if $2+3i$ is a solution, then $2-3i$ is a solution

conj. pairs – change the sign on the imag. part

ex: $3 - 5i \to 3 + 5i$

ex: $6 - \sqrt{7}i \to 6 + \sqrt{7}i$

ex: $6 + 7\sqrt{2}i \to 6 - 7\sqrt{2}i$

ex: $-5i \to 5i$

ex: $6 \to 6$ (no conj. pair – no imag. part)

ex: $-8i + 5 \to 5 + 8i$