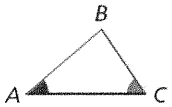
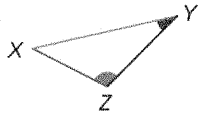
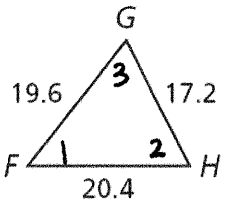
II. Relationships in Triangles

Theorems Angle-Side Relationships in Triangles

THEOREM	HYPOTHESIS	CONCLUSION
5-5-1 If two sides of a triangle are not congruent, then the larger angle is opposite the longer side. (In △, larger ∠ is opp. longer side.)	 $AB > BC$	$m\angle C > m\angle A$
5-5-2 If two angles of a triangle are not congruent, then the longer side is opposite the larger angle. (In △, longer side is opp. larger ∠.)	 $m\angle Z > m\angle Y$	$XY > XZ$

Example 3: Write the angles in order from smallest to largest.

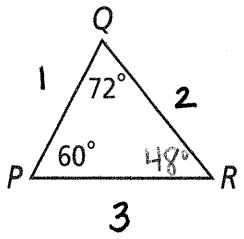


$\angle F, \angle H, \angle G$

(17.2) (19.6) (20.4)

$\therefore m\angle F < m\angle H < m\angle G$

Example 4: Write the sides in order from shortest to longest.



$\overline{PQ}, \overline{QR}, \overline{PR}$

$\therefore PQ < QR < PR$

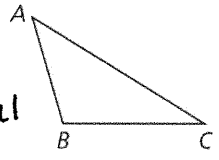
III. Triangle Inequality

Theorem 5-5-3 Triangle Inequality Theorem

The sum of any two side lengths of a triangle is greater than the third side length.

$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$

strictly greater than NOT equal to!!



Example 5: Tell whether a triangle can have sides with the given lengths. Explain.

a. 7, 10, 19

$7+10 > 19$

$17 > 19$ False!

No, can't be a △!

b. 2.3, 3.1, 4.6

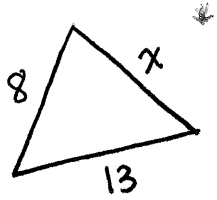
yes, can be a △!

$2.3+3.1 > 4.6 \Rightarrow 5.4 > 4.6$

$2.3+4.6 > 3.1 \Rightarrow 6.9 > 3.1$

$3.1+4.6 > 2.3 \Rightarrow 7.7 > 2.3$

Example 6: The lengths of two sides of a triangle are 8 inches and 13 inches.
Find the range of possible lengths for the third side. Let x represent the 3rd side.



$8+13>x$

$21>x$

$x<21$

$8+x>13$

$x>5$

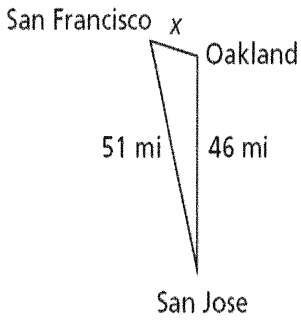
$13+x>8$

$x>-5$

no negative side lengths!

$5\text{in} < x < 21\text{in}$

Example 7: The figure shows the approximate distances between cities in California.
What is the range of distances from San Francisco to Oakland?



$51+x>46$

$x>-5$

$51+46>x$

$97>x$

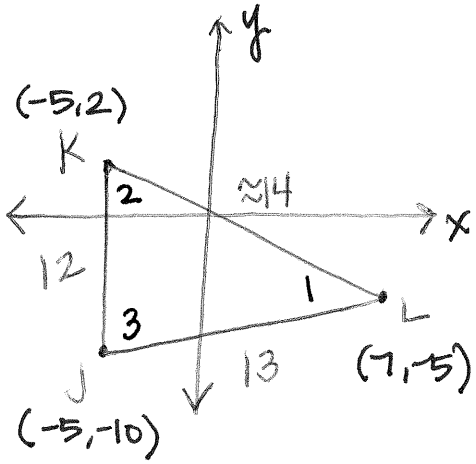
$46+x>51$

$x>5$

no negative side lengths!

$5\text{mi} < x < 97\text{mi}$

Example 8: List the angles of ΔJKL in order from smallest to largest.



$J(-5,-10)\ K(-5,2)\ L(7,-5)$

$JK = \sqrt{(-5-(-5))^2+(2-(-10))^2} = \sqrt{0^2+(12)^2} = \sqrt{144} = 12$

$JL = \sqrt{(-5-(-10))^2+(7-(-5))^2} = \sqrt{(5)^2+(12)^2} = \sqrt{169} = 13$

$KL = \sqrt{(-5-2)^2+(7-(-5))^2} = \sqrt{(-7)^2+(12)^2} = \sqrt{49+144}$

$= \sqrt{193}$

≈ 14

$m<L < m<K < m<J$

$\angle L, \angle K, \angle J$