

Section 6.2 - Multiplying Polynomials

Focus - 1) Given the polynomial: $y = 8x^7 - 14x^{10} + 2x - 9$

of terms: 4

Std. Form: $y = -14x^{10} + 8x^7 + 2x - 9$

Lead coeff: -14

Degree: 10

2) Find the degree of $9x^2y^8 + 4x^4y^{10}$

Degree: 14

Multiplying Polynomials

1) $(6x^4)(2x^7) = 12x^{11}$

2) $(3x^4)^3 = 27x^{12}$

3) $(2x^3)^2 + (3x^2)^3 = 4x^6 + 27x^6$ (cant add the terms - exp. do not match)

4) $6x^4(2x^2 - 5x + 7) = 12x^6 - 30x^5 + 42x^4$

5) $(2x - 3y)^2 \rightarrow$ FOIL

$$(2x - 3y)(2x - 3y)$$

$$4x^2 - 6xy - 6yx + 9y^2$$

$$4x^2 - 12xy + 9y^2$$

6) $(x^2 - 6x + 9)(4x^4 - 3x^2 + 2x)$

$$4x^6 - 3x^4 + 2x^3 - 24x^5 + 18x^3 - 12x^2 + 36x^4 - 27x^2 + 18x$$

$$4x^6 - 24x^5 + 33x^4 + 20x^3 - 39x^2 + 18x$$

Binomial Expansion - Pascal's Triangle

In terms of Addition

| | | | | | | | | |
|---|---|----|----|----|----|----|---|---|
| 1 | | | | | | | | |
| 1 | 1 | | | | | | | |
| 1 | 2 | 1 | | | | | | |
| 1 | 3 | 3 | 1 | | | | | |
| 1 | 4 | 6 | 4 | 1 | | | | |
| 1 | 5 | 10 | 10 | 5 | 1 | | | |
| 1 | 6 | 15 | 20 | 15 | 6 | 1 | | |
| 1 | 7 | 21 | 35 | 35 | 21 | 7 | | |
| 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 |

$$\begin{aligned} (x+y)^1 &= x+y \\ (x+y)^2 &= x^2+2xy+y^2 \\ (x+y)^3 &= x^3+3x^2y+3xy^2+y^3 \\ (x+y)^4 &= x^4+4x^3y+6x^2y^2+4xy^3+y^4 \end{aligned}$$

- 1) There are always $n+1$ terms
- 2) The coeffs of each term come from Pascal's Δ

- 3) $x \rightarrow$ power is going to decrease by one as you move to the right
- 4) $y \rightarrow$ power is going to increase by one as you move to the right
- 5) Sum of the exp in any term is always "n"

ex: $(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

ex: $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

ex: $(2x+3y)^4 = (A+B)^4 = A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$

\downarrow \downarrow
 A B

$$= (2x)^4 + 4(2x)^3(3y) + 6(2x)^2(3y)^2 + 4(2x)(3y)^3 + (3y)^4$$

$$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$$

ex: $(2x^2-y)^6 = (A+B)^6 = A^6 + 6A^5B + 15A^4B^2 + 20A^3B^3 + 15A^2B^4 + 6AB^5 + B^6$

\downarrow \downarrow
 A B

A w/ sub,
signs alternate
-/+

$$= (2x^2)^6 + 6(2x^2)^5(-y) + 15(2x^2)^4(-y)^2 + 20(2x^2)^3(-y)^3 + 15(2x^2)^2(-y)^4 + 6(2x^2)(-y)^5 + (-y)^6$$

$$= 64x^{12} - 192x^{10}y + 240x^8y^2 - 160x^6y^3 + 60x^4y^4 - 12x^2y^5 + y^6$$

ex: $(x-y)^3 = (A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

\downarrow \downarrow
 A B

$$= x^3 + 3(x^2)(-y) + 3(x)(-y)^2 + (-y)^3$$

$$= x^3 - 3x^2y + 3xy^2 - y^3$$