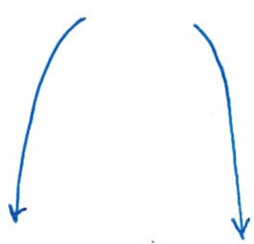
Section 6.7 - Sketching Polynomials:

1) <u>Degree</u> → highest power

Even

Leading coeff > 0 (+)

OR

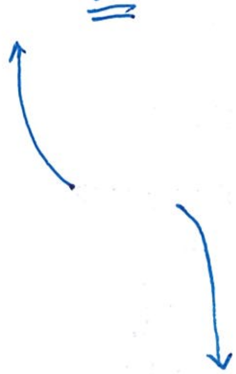


Leading coeff < 0 (−)

Odd

Leading coeff > 0 (+)

OR



Leading coeff < 0 (−)

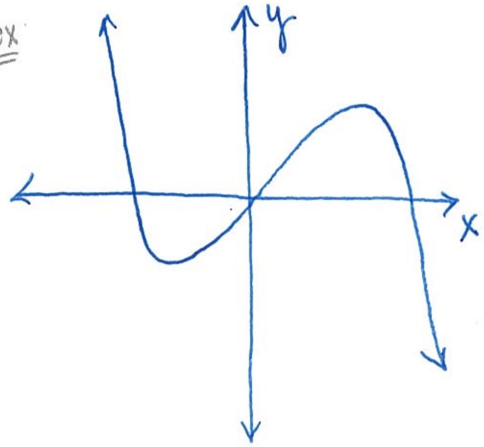
Extreme Behavior "the arms of the function"

2) <u>multiplicity</u> → Repeated zero (power on the factor)

- If mult is <u>even</u> → function will touch the x axis at the zero.
- If mult. is <u>odd</u> → function goes through the x axis at the zero.

3) Describing the extreme behavior

ex:



as $x \to -\infty$, $f(x) \to \infty$

as $x \to \infty$, $f(x) \to -\infty$

ex: $P(x) = -4x^4 - 3x^3 + x^2 + 4$

$x \to +\infty$, $P(x) \to -\infty$

$x \to -\infty$, $P(x) \to -\infty$

ex: $P(x) = x^5 - 4x^2 + 3x - 1$

$x \to -\infty$, $P(x) \to -\infty$

$x \to \infty$, $P(x) \to \infty$

4) Examples: (NO SHARP POINTS!)

ex: $y = -18x^3 - 15x^2 + 12x$

Deg: $3 \to$ odd

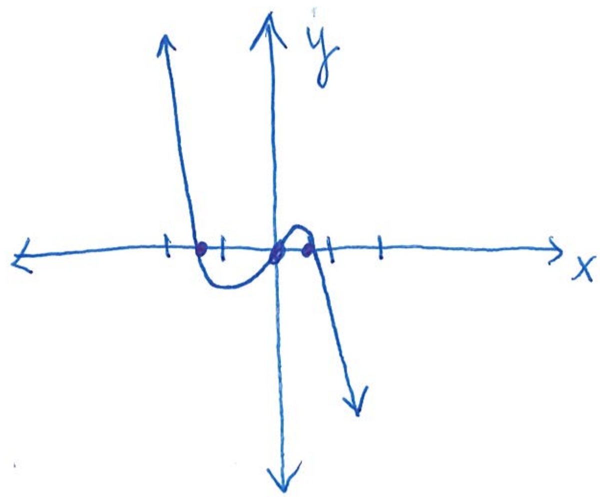
LC: $-18 \to$ neg. ↑ ↓

Zeros: $-3x(6x^2+5x-4) = 0$

$-3x(3x+4)(2x-1) = 0$

$x = 0$ (1) $\quad x = -\frac{4}{3}$ (1) $\quad x = \frac{1}{2}$ (1)

mult. 1 → through x axis



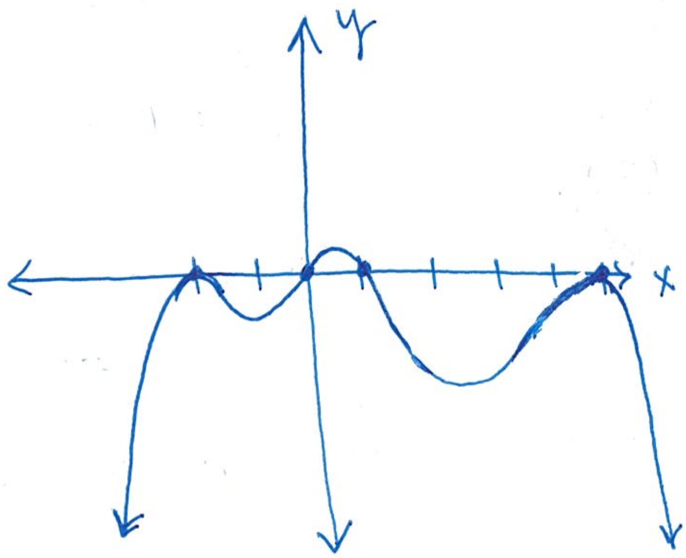
ex: $y = -3x(x-1)^3(x+2)^2(x-5)^4 = 0$

Deg: 10 → even

LC: −3 → neg ↙ ↘

Zeros: $x=0$, $x=1$, $x=-2$, $x=5$

(1) through, (3) through, (2) touch, (4) touch



ex: $y = -3x^2(x-1)^3(x+5)^4$

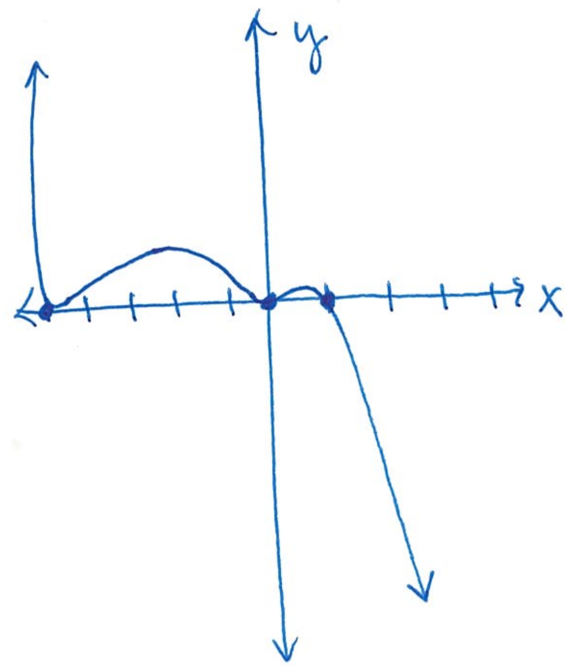
Deg: 9 → odd

LC: −3 → neg ↖ ↘

Zeros:

$x=0$, $x=1$, $x=-5$

(2) touch, (3) through, (4) touch



ex: $y = x^3 - 4x^2 + 4x$

Deg: 3 → odd

LC: 1 → positive

Zeros:

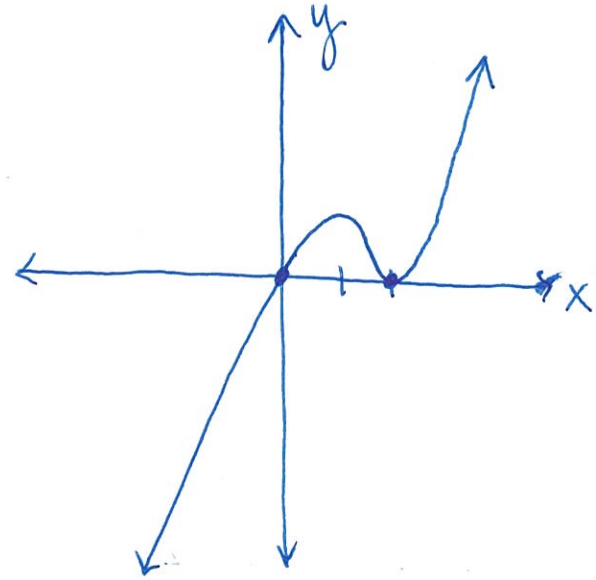
$$0 = x^3 - 4x^2 + 4x$$

$$0 = x(x^2 - 4x + 4)$$

$$0 = x(x-2)^2$$

$x = 0$, $x = 2$

(1) through   (2) touch



Why might you not see all of the roots on your graph?

1) multiplicity

2) imaginary roots - we care about these w/ solving not w/ graphing