Section 8.6 Radical Expressions and Rational Exponents
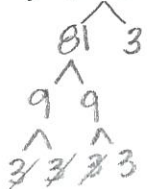
Focus: Simplify...

1) $\sqrt{147} = \sqrt{49} \cdot \sqrt{3} = 7\sqrt{3}$
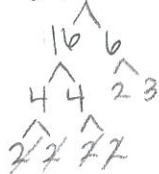
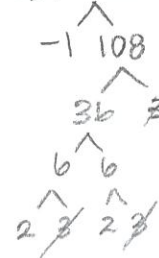2) $\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$

Simplifying Radicals...

1) $\sqrt[3]{243} = 3\sqrt[3]{9}$

2) $\sqrt[4]{96} = 2\sqrt[4]{6}$

3) $\sqrt[3]{-108} = -3\sqrt[3]{4}$

Simplifying Radicals with Variables...

1) $\sqrt{x^2 y^4 z^6 a^7}$

$x \cdot y^2 \cdot z^3 \cdot a^3 \sqrt{a}$

Even - Even - odd $\Rightarrow$ abs value!

↑ Index   ↑ exp inside   ↑ exp outside

Could have a hidden negative for variables!

$|x| y^2 |z^3| a^3 \sqrt{a}$

$|x z^3| y^2 a^3 \sqrt{a}$

2) $\sqrt[3]{x^5 y^6 z^{10}}$
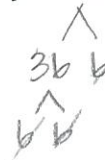
$x y^2 z^3 \sqrt[3]{x^2 z}$

no abs. value needed

EEO

3) $\sqrt[4]{32 x^8 y^6 z^3 a^4}$
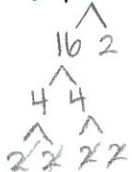
$2x^2 y a \sqrt[4]{2y^2 z^3}$

$2x^2 |ya| \sqrt[4]{2y^2 z^3}$

4) $\sqrt[3]{216 a^9 b^{32} c^{77}}$

$6 a^3 b^{10} c^{25} \sqrt[3]{b^2 c^2}$

no abs. value needed

EEO

Properties of Roots...

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

1) $\sqrt[3]{2x} \cdot \sqrt[3]{12x^2} = \sqrt[3]{24x^3} = 2x\sqrt[3]{3}$

4 6
2 x x 3

1) $\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$

2) $\sqrt[3]{x^7} \cdot \sqrt[3]{x^2} = \sqrt[3]{x^9} = x^3$

2) $\sqrt{\frac{5}{4x}} = \frac{\sqrt{5}}{\sqrt{4x}} = \frac{\sqrt{5}}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{5x}}{2\sqrt{x^2}} = \frac{\sqrt{5}}{2|x|}$

3) $\sqrt{2x} \cdot \sqrt[3]{4xy} =$ cant be done
diff. index.

$\sqrt{4x^2} \cdot \sqrt[3]{8xy} \Rightarrow 2|x| \cdot 2\sqrt[3]{xy}$
$4|x|\sqrt[3]{xy}$

3) $\sqrt[3]{\frac{x^3}{7}} = \frac{\sqrt[3]{x^3}}{\sqrt[3]{7}} = \frac{x}{\sqrt[3]{7}} \cdot \frac{\sqrt[3]{49}}{\sqrt[3]{49}} = \frac{x\sqrt[3]{49}}{\sqrt[3]{343}}$
$= \frac{x\sqrt[3]{49}}{7}$

Rewriting Rational Exponents as Radical Expressions

$$a^{\frac{x}{y}} = (\sqrt[y]{a})^x$$

1) $2^{\frac{2}{3}} = \left(\sqrt[3]{2}\right)^2$

2) $10^{-\frac{7}{4}} = \frac{1}{\left(\sqrt[4]{10}\right)^7}$

3) $(6x)^{\frac{5}{3}} = \left(\sqrt[3]{6x}\right)^5$

Rewriting Radical Expressions as Rational Exponents

1) $\sqrt[2]{7}^1 = 7^{1/2}$

2) $\left(\sqrt[6]{10}\right)^5 = 10^{5/6}$

3) $\left(\sqrt[6]{2b}\right)^7 = (2b)^{7/6}$

4) $\frac{1}{\left(\sqrt[3]{4z}\right)^2} = \frac{1}{(4z)^{2/1}}$

Simplify...

1) $81^{\frac{3}{4}}$

$\left(\sqrt[4]{81}\right)^3$

9 9
3 3 3 3
$(3)^3 = 27$

2) $144^{-\frac{1}{2}}$

$\frac{1}{\sqrt{144}} = \frac{1}{12}$

3) $216^{-\frac{5}{3}}$

$\frac{1}{\left(\sqrt[3]{216}\right)^5} = \frac{1}{(6)^5}$

36 6
6 6

$= \frac{1}{7776}$

4) $(27x^9)^{-\frac{1}{3}}$

$= \frac{1}{\sqrt[3]{27x^9}}$

$= \frac{1}{3x^3}$

5) $(1000n^6)^{\frac{5}{3}}$

$= \left(\sqrt[3]{1000n^6}\right)^5$

$= (10n^2)^5$

$= 100000n^{10}$