

Algebra II Honors
Solving Radical Equations

I. Solving Equations with Single Radicals

Steps to Solving a Radical Equation with ONE Radical

1. Isolate the radical. (Get the radical by itself!)
2. Square both sides/Cube both sides/etc (Get rid of the radical!)
3. Solve.
4. Check for extraneous solutions. (What are these again?)

Dxy 1

* Example #1: $5 - \sqrt{x-1} = 2$

$$-\sqrt{x-1} = -3$$

$$\sqrt{x-1} = 3$$

$$(\sqrt{x-1})^2 = 3^2$$

$$x-1 = 9$$

$$x = 10 \checkmark$$

Example #4: $5\sqrt[3]{4x+3} = 15$

$$\sqrt[3]{4x+3} = 3$$

$$(\sqrt[3]{4x+3})^3 = 3^3$$

$$4x+3 = 27$$

$$4x = 24$$

$$x = 6 \checkmark$$

Dxy 1

* Example #4: $\sqrt{x^2-9} = x+1$

$$x^2-9 = (x+1)^2$$

$$x^2-9 = x^2+2x+1$$

$$-9 = 2x+1$$

$$-10 = 2x$$

$$x = -5$$

no solution!

Focus
Dxy 2

* Example #6: $3\sqrt{2x+2} = 2x-2$

$$(3\sqrt{2x+2})^2 = (2x-2)^2$$

$$9(2x+2) = 4x^2-8x+4$$

$$18x+18 = 4x^2-8x+4$$

$$0 = 4x^2-26x-14$$

$$0 = 2(2x^2-13x-7)$$

$$0 = 2(2x+1)(x-7)$$

$$x = -\frac{1}{2}, x = 7 \checkmark$$

Example #3: $\sqrt{2x+5} - x = 3$

$$\sqrt{2x+5} = x+3$$

$$2x+5 = (x+3)^2$$

$$2x+5 = x^2+6x+9$$

$$0 = x^2+4x+4$$

$$0 = (x+2)(x+2)$$

$$0 = (x+2)^2$$

$$x = -2 \checkmark$$

Dxy 1

* Example #5: $2x = 3\sqrt{x+3} + 3$

$$2x-3 = 3\sqrt{x+3}$$

$$(2x-3)^2 = (3\sqrt{x+3})^2$$

$$4x^2-12x+9 = 9(x+3)$$

$$4x^2-12x+9 = 9x+27$$

$$4x^2-21x-18 = 0$$

$$(4x+3)(x-6) = 0$$

$$x = -\frac{3}{4}, x = 6 \checkmark$$

Example #7: $\sqrt{1-4x} - x = 5$

$$\sqrt{1-4x} = x+5$$

$$1-4x = (x+5)^2$$

$$1-4x = x^2+10x+25$$

$$0 = x^2+14x+24$$

$$0 = (x+12)(x+2)$$

$$x = -12, x = -2$$

Day 2
Example #8: $\frac{\sqrt{5+x}}{2} - x - 2 = 0$

$$\frac{\sqrt{5+x}}{2} = x+2$$

$$\sqrt{5+x} = 2x+4$$

$$5+x = (2x+4)^2$$

$$5+x = 4x^2+16x+16$$

$$0 = 4x^2+15x+11$$

$$0 = (4x+11)(x+1)$$

$$x = -11/4, x = -1$$

II. Solving Equations with Double Radicals

Steps to Solving a Radical Equation with DOUBLE Radicals

1. Separate the Radicals - Make sure there is ONE radical per side!
2. Square both sides.
3. Turns into a single radical problem!
4. Solve
5. Check for extraneous solutions.

Day 1
Example #1: $(\sqrt{-2-n})^2 = (\sqrt{3n+38})^2$

$$-2-n = 3n+38$$

$$-2 = 4n+38$$

$$-40 = 4n$$

$$n = -10 \checkmark$$

Day 1
Example #2: $(\sqrt{x+6})^2 = (\sqrt{x+3}+1)^2$

$$x+6 = x+3 + \sqrt{x+3} + \sqrt{x+3} + 1$$

$$x+6 = x+4 + 2\sqrt{x+3}$$

$$2 = 2\sqrt{x+3}$$

$$1 = \sqrt{x+3}$$

$$1 = x+3$$

$$x = -2 \checkmark$$

Example #3: $(2\sqrt{x})^2 = (\sqrt{4x-3}+1)^2$

$$4x = 4x-3 + 2\sqrt{4x-3} + 1$$

$$4x = 4x-2 + 2\sqrt{4x-3}$$

$$0 = -2 + 2\sqrt{4x-3}$$

$$2 = 2\sqrt{4x-3}$$

$$1 = \sqrt{4x-3}$$

$$1 = 4x-3$$

$$4 = 4x$$

$$x = 1 \checkmark$$

Example #4: $(2\sqrt{x})^2 = (\sqrt{x-1}+2)^2$

$$4x = x-1 + 4\sqrt{x-1} + 4$$

$$4x = x+3 + 4\sqrt{x-1}$$

$$(3x-3)^2 = (4\sqrt{x-1})^2$$

$$9x^2-18x+9 = 16(x-1)$$

$$9x^2-18x+9 = 16x-16$$

$$9x^2-34x+25 = 0$$

$$(9x-25)(x-1) = 0$$

$$x = \frac{25}{9}, x = 1 \checkmark$$

Example #5: $\sqrt{x+2} - \sqrt{x-3} = 1$

$$(\sqrt{x+2})^2 = (1 + \sqrt{x-3})^2$$

$$x+2 = 1 + 2\sqrt{x-3} + x-3$$

$$x+2 = -2 + x + 2\sqrt{x-3}$$

$$4 = 2\sqrt{x-3}$$

$$2 = \sqrt{x-3}$$

$$4 = x-3$$

$x = 7 \checkmark$

Example #7: $\sqrt{5-2x} + \sqrt{x-1} = 2$

$$\sqrt{5-2x} = 2 - \sqrt{x-1}$$

$$5-2x = 4 - 4\sqrt{x-1} + x-1$$

$$5-2x = 3+x-4\sqrt{x-1}$$

$$(2-3x)^2 = (-4\sqrt{x-1})^2$$

$$4-12x+9x^2 = 16(x-1)$$

$$9x^2-12x+4 = 16x-16$$

$$9x^2-28x+20 = 0$$

$$(9x-10)(x-2) = 0$$

$x = \frac{10}{9}, x = 2$

Example #9: $\sqrt{x^2+5} + \sqrt{5-x^2} = 4$

Day 2

$$\sqrt{x^2+5} = 4 - \sqrt{5-x^2}$$

$$x^2+5 = 16 - 8\sqrt{5-x^2} + 5-x^2$$

$$x^2+5 = 21 - x^2 - 8\sqrt{5-x^2}$$

$$(2x^2-16)^2 = (8\sqrt{5-x^2})^2$$

$$4x^4 - 64x^2 + 256 = 64(5-x^2)$$

$$4x^4 - 64x^2 + 256 = 320 - 64x^2$$

$$4x^4 + 256 = 320$$

$$4x^4 = 64$$

$x^4 = 16$
 $x = \pm 2 \checkmark$

Day 1

Example #6: $\sqrt{7+x} + \sqrt{6-x} = 5$

$$\sqrt{7+x} = 5 - \sqrt{6-x}$$

$$7+x = 25 - 10\sqrt{6-x} + 6-x$$

$$7+x = 31 - x - 10\sqrt{6-x}$$

$$(-24+2x)^2 = (10\sqrt{6-x})^2$$

$$576 - 96x + 4x^2 = 100(6-x)$$

$$4x^2 - 96x + 576 = 600 - 100x$$

Focus Day 2

$$4x^2 + 4x - 24 = 0 \rightarrow 4(x^2 + x - 6) = 0$$

Example #8: $\sqrt{2x+1} + \sqrt{x-3} = 2$ $4(x+3)(x-2) = 0$

$$\sqrt{2x+1} = 2 - \sqrt{x-3}$$

$$2x+1 = 4 - 4\sqrt{x-3} + x-3$$

$$2x+1 = 1+x-4\sqrt{x-3}$$

$$x = -4\sqrt{x-3}$$

$$x^2 = 16(x-3)$$

$$x^2 = 16x - 48$$

$$x^2 - 16x + 48 = 0$$

$$(x-12)(x-4) = 0$$

$x = 12, x = 4$
no solution

Day 2

III. Solving Equations with Rational Exponents

1. $(3x-1)^{\frac{1}{5}} = 2$ $\sqrt[5]{3x-1} = 2$

$$3x-1 = 32 \quad x = 11$$

$$3x = 33$$

3. $(x+5)^{\frac{1}{3}} = 3$ $\sqrt[3]{x+5} = 3$

$$x+5 = 27$$

$$x = 22$$

5. $3(x+6)^{\frac{1}{2}} = 9$ $3\sqrt{x+6} = 9$

$$\sqrt{x+6} = 3 \quad x = 3$$

$$x+6 = 9$$

2. $x = (x+12)^{\frac{1}{2}}$ $x = \sqrt{x+12}$ $(x-4)(x+3) = 0$

$$x^2 = x+12 \quad x = 4$$

$$x^2 - x - 12 = 0 \quad x = -3$$

4. $(2x+15)^{\frac{1}{2}} = x$ $\sqrt{2x+15} = x$

$$2x+15 = x^2$$

$$0 = x^2 - 2x - 15 \quad x = 5$$

$$0 = (x-5)(x+3) \quad x = -3$$

6. $(45-9x)^{\frac{1}{2}} = x-5$

$$\sqrt{45-9x} = x-5$$

$$45-9x = x^2 - 10x + 25$$

$$0 = x^2 - 10x + 25 \quad (x-5)(x+4) = 0$$

$$x = 5, x = -4$$