Studyguide (verify identities) Key

1.) $\tan(\pi/2 - x)\sec x = \boxed{\csc x}$

$$\downarrow$$

$$(\cot x)(\sec x)$$

$$\downarrow \quad \downarrow$$

$$\frac{\cos \cancel{x}}{\sin x} \cdot \frac{1}{\cos \cancel{x}} = \frac{1}{\sin x} = \underline{\underline{\csc x}}$$

2.) $\dfrac{\cos(\pi/2 - x)}{\sin(\pi/2 - x)} = \boxed{\tan x}$

$$\frac{\sin x}{\cos x} = \underline{\underline{\tan x}}$$

3.) $\dfrac{\csc(-x)}{\sec(-x)} = \boxed{-\cot x}$

$$\frac{-\csc x}{\sec x} = \frac{-\frac{1}{\sin x}}{\frac{1}{\cos x}} = \frac{-1}{\sin x} \cdot \frac{\cos x}{1} = \frac{-\cos x}{\sin x} = -\underline{\cot x}$$

4.) $(1 + \sin y)(1 + \sin(-y)) = \boxed{\cos^2 y}$

FOIL

$= 1 - \cancel{\sin y} + \cancel{\sin y} - \sin^2 y$

$= 1 - \sin^2 y$ (Pyth. identity $\rightarrow$ $\begin{array}{l} \sin^2 \cancel{x} + \cos^2 x = 1 \\ -\sin^2 x \qquad\quad -\sin^2 x \\ \hline \cos^2 x = 1 - \sin^2 x \end{array}$

$= \underline{\cos^2 y}$

5.) $\dfrac{\cos(-x)}{1+\sin(-x)} = \boxed{\sec x + \tan x}$

multiply by complex conjugate

$$\frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x + \cos x \sin x}{1 + \cancel{\sin x} - \cancel{\sin x} - \sin^2 x}$$

$$= \frac{\cos x + \cos x \sin x}{1 - \sin^2 x}$$

$$= \frac{\cos x + \cos x \sin x}{\cos^2 x}$$

$$= \frac{\cos x}{\cos^2 x} + \frac{\cos x \sin x}{\cos^2 x}$$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \underline{\underline{\sec x + \tan x}}$$

6.) $\dfrac{\tan x \cot x}{\cos x} = \boxed{\sec x}$

$$= \frac{\cancel{\tan x \cdot \frac{1}{\tan x}}}{\cos x}$$

$$= \frac{1}{\cos x} = \underline{\underline{\sec x}}$$

7.) $\dfrac{1+\csc\theta}{\sec\theta} - \cot\theta = \boxed{\cos\theta}$

$$= \frac{1}{\sec\theta} + \frac{\csc\theta}{\sec\theta} - \cot\theta$$

$$= \cos\theta + \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} - \cot\theta$$

$$= \cos\theta + \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{1} - \cot\theta$$

$$= \cos\theta + \frac{\cos\theta}{\sin\theta} - \cot\theta$$

$$= \cos\theta + \cancel{\cot\theta} - \cancel{\cot\theta}$$

$$= \underline{\underline{\cos\theta}}$$

8.) $\csc^4 x - 2\csc^2 x + 1 = \boxed{\cot^4 x}$

$$x^4 - 2x^2 + 1$$
$$(x^2-1)(x^2-1)$$

$$= (\csc^2 x - 1)(\csc^2 x - 1)$$

$$= (\cot^2 x)(\cot^2 x) = \underline{\underline{\cot^4 x}}$$

9.) $\dfrac{\sin x}{1-\cos x} = \boxed{\dfrac{1+\cos x}{\sin x}}$

multiply by complex conjugate

$$= \frac{\sin x}{(1-\cos x)} \cdot \frac{1+\cos x}{(1+\cos x)} = \frac{\sin x + \sin x\cos x}{1+\cancel{\cos x}-\cancel{\cos x}-\cos^2 x}$$

$$= \frac{\sin x + \sin x\cos x}{1-\cos^2 x}$$

$$= \frac{\sin x(1+\cos x)}{\sin^2 x}$$

$$= \frac{1+\cos x}{\sin x}$$

10.) $\dfrac{\cot x}{\csc x - 1} = \boxed{\dfrac{\csc x + 1}{\cot x}}$

multiply by complex conjugate

$$= \frac{\cot x}{\csc x - 1} \cdot \frac{\csc x + 1}{\csc x + 1} = \frac{\cot x\csc x + \cot x}{\csc^2 x + \cancel{\csc x} - \cancel{\csc x} - 1}$$

$$= \frac{\cot x\csc x + \cot x}{\csc^2 x - 1}$$

$$= \frac{\cot x(\csc x + 1)}{\cot^2 x}$$

$$= \frac{\csc x + 1}{\cot x}$$