Key

Name : ______________________________

For numbers 1-3, use the ***sum/difference*** formulas to evaluate:

$150^\circ + 45^\circ$

1) $\sin(195°) = \sin 150 \cos 45 + \cos 150 \sin 45$

$= \left(\tfrac{1}{2}\right)\left(\tfrac{\sqrt{2}}{2}\right) + \left(-\tfrac{\sqrt{3}}{2}\right)\left(\tfrac{\sqrt{2}}{2}\right)$

$= \dfrac{\sqrt{2}}{4} + \dfrac{-\sqrt{6}}{4} = \boxed{\dfrac{\sqrt{2}-\sqrt{6}}{4}}$

2) $\cos(\frac{5\pi}{12}) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$\dfrac{3\pi}{12} + \dfrac{2\pi}{12}$ $\quad \left(\tfrac{\sqrt{2}}{2}\right)\left(\tfrac{\sqrt{3}}{2}\right) - \left(\tfrac{\sqrt{2}}{2}\right)\left(\tfrac{1}{2}\right)$

$\dfrac{\pi}{4} + \dfrac{\pi}{6}$ $\quad = \dfrac{\sqrt{6}}{4} - \dfrac{\sqrt{2}}{4} = \boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

3) $\tan(\frac{-7\pi}{12}) =$

$-\dfrac{3\pi}{12} - \dfrac{4\pi}{12}$ $\quad \dfrac{\tan\frac{-\pi}{4} - \tan\frac{\pi}{3}}{1+\tan(-\pi/4)(\tan\pi/3)} = \dfrac{-1-\sqrt{3}}{1+(-1)(\sqrt{3})} = \dfrac{-1-\sqrt{3}}{1-\sqrt{3}}$

$-\dfrac{\pi}{4} - \dfrac{\pi}{3}$

For numbers 4 and 5, use the ***half angle*** formulas to evaluate:

Q1

4) $\sin(15°) = +\sqrt{\dfrac{1-\cos 30}{2}} = \sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\dfrac{\frac{2-\sqrt{3}}{2}}{2}}$

$30/2$

$= \boxed{\sqrt{\dfrac{2-\sqrt{3}}{4}}}$

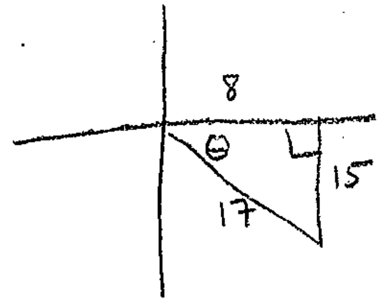
Q2

5) $\cos(112.5°) = -\sqrt{\dfrac{1+\cos 225}{2}} = -\sqrt{\dfrac{1+\frac{-\sqrt{2}}{2}}{2}} = -\sqrt{\dfrac{\frac{2-\sqrt{2}}{2}}{2}}$

$225/2$

$= \boxed{-\sqrt{\dfrac{2-\sqrt{2}}{4}}}$

$\cos\theta = \dfrac{8}{17} \qquad {}^{3\pi}\!/_{2} < \theta < 2\pi \ (Q4)$



1. $\sec 2\theta =$

flip $\cos 2\theta$ → $2\cos^2\theta - 1$

$2\left(\frac{8}{17}\right)^2 - 1 = 2\left(\frac{64}{289}\right) - 1$

$= \frac{128}{289} - \frac{289}{289} = \frac{-161}{289}$

$\boxed{\sec 2\theta = {-289}/_{161}}$

2. $\cot 2\theta =$

flip $\tan 2\theta$ → $\dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\left(-{}^{15}\!/_{8}\right)}{1-\left(-{}^{15}\!/_{8}\right)^2} = \dfrac{\frac{-30}{8}}{\frac{64}{64} - \frac{225}{64}} = \dfrac{\frac{-30}{8}}{\frac{-161}{64}} = \frac{-30}{\cancel{8}_1} \times \frac{\cancel{64}^{8}}{161}$

$= \frac{-240}{-161} = \frac{240}{161}$

$\boxed{\cot 2\theta = {161}/_{240}}$

3. $\csc\dfrac{\theta}{2} =$

flip $\sin{}^{\theta}\!/_{2}$ → $+\sqrt{\dfrac{1-\left({}^{8}\!/_{17}\right)}{2}}$

$= +\sqrt{\dfrac{\frac{17}{17} - \frac{8}{17}}{2}} = +\sqrt{\dfrac{\frac{9}{17}}{2}} = +\sqrt{\frac{9}{17}\cdot\frac{1}{2}} = +\sqrt{\frac{9}{34}} = \boxed{+\sqrt{{34}/_{9}}}$

θ is between 270° & 360° so ½θ b/w 135° & 180° → Q2 (sin/csc +)

4. $\csc 2\theta =$

flip $\sin 2\theta$ → $2(\sin\theta)(\cos\theta)$

$\frac{2}{1}\left(-\frac{5}{17}\right)\left(\frac{8}{17}\right) = \frac{-240}{289} = \boxed{\frac{-289}{240}}$

5. $\sec\dfrac{\theta}{2} =$

flip $\cos\dfrac{\theta}{2}$ → $-\sqrt{\dfrac{1+\cos\theta}{2}} = -\sqrt{\dfrac{1+{}^{8}\!/_{17}}{2}} = -\sqrt{\dfrac{\frac{17}{17}+\frac{8}{17}}{2}} = -\sqrt{\dfrac{\frac{25}{17}}{2}} = -\sqrt{\frac{25}{17}\cdot\frac{1}{2}}$

Q2 → cos/sec (-)

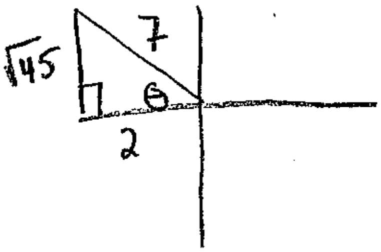
$= -\sqrt{\frac{25}{34}}$

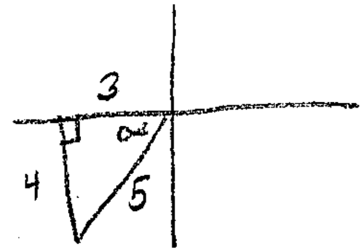
$= \boxed{-\sqrt{{34}/_{25}}}$

6. $\cot\dfrac{\theta}{2} =$

flip $\tan{}^{\theta}\!/_{2}$ → $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{{}^{17}\!/_{17}\,\cancel{1} - \frac{8}{17}}{\frac{-15}{17}} = \frac{9}{17} \times -\frac{17}{15} = \frac{-9}{15} = \boxed{-{15}/_{9}}$

6) $\cos\theta = -\frac{2}{7}$ $\quad \pi/2 < \theta < \pi$ (Q2) $\qquad \tan\alpha = \frac{4}{3}$ $\quad \pi < \alpha < \frac{3\pi}{2}$ (Q3)





a) $\sin\left(\frac{\theta}{2}\right) = +\sqrt{\frac{1-\cos\theta}{2}}$

$+\sqrt{\frac{1-(-2/7)}{2}}$

$= \sqrt{\frac{1+2/7}{2}} = \sqrt{\frac{\frac{7}{7}+\frac{2}{7}}{2}} = \sqrt{\frac{9}{7}\times\frac{1}{2}} = \boxed{\sqrt{9/14}}$

b) $\cos(2\alpha) = 2\cos^2\alpha - 1$

$2\left(\frac{-3}{5}\right)^2 - 1$

$2\left(\frac{9}{25}\right) - 1$

$\frac{18}{25} - 1 = \frac{18}{25} - \frac{25}{25}$

$= \boxed{-7/25}$

c) $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{\cancel{2}\left(\frac{-\sqrt{45}}{\cancel{2}}\right)}{1-\left(\frac{-\sqrt{45}}{2}\right)^2} = \frac{-\sqrt{45}}{1-\frac{45}{4}}$

$= \frac{-\sqrt{45}}{\frac{4}{4}-\frac{45}{4}} = \frac{-\sqrt{45}}{\frac{-41}{4}} = \frac{-\sqrt{45}}{1}\cdot\frac{-4}{41} = \frac{4\sqrt{45}}{41} = \frac{4\cdot\sqrt{9}\cdot\sqrt{5}}{41} = \boxed{\frac{12\sqrt{5}}{41}}$

d) $\cos(\alpha-\theta) =$

$\cos\alpha\cos\theta + \sin\alpha\sin\theta$

$\left(\frac{-3}{5}\right)\left(\frac{-2}{7}\right) + \left(\frac{-4}{5}\right)\left(\frac{\sqrt{45}}{7}\right)$

$= \frac{6}{35} - \frac{4\sqrt{45}}{35} = \frac{6-4\sqrt{45}}{35} = \boxed{\frac{6-12\sqrt{45}}{35}}$

e) $\sin(\theta+\alpha) =$

$\sin\theta\cos\alpha + \cos\theta\sin\alpha$

$\left(\frac{\sqrt{45}}{7}\right)\left(\frac{-3}{5}\right) + \left(\frac{-2}{7}\right)\left(\frac{-4}{5}\right)$

$= \frac{-3\sqrt{45}}{35} + \frac{8}{35} = \frac{-3\cdot3\cdot\sqrt{5}}{35} + \frac{8}{35}$

$= \boxed{\frac{-9\sqrt{5}+8}{35}}$

f) $\tan(\alpha+\theta) =$

$\frac{\tan\alpha + \tan\theta}{1-\tan\alpha\tan\theta}$

$= \frac{\frac{4}{3} + \frac{-\sqrt{45}}{2}}{1-\left(\frac{4}{3}\right)\left(-\frac{\sqrt{45}}{2}\right)}$

$= \frac{\frac{8}{6} - \frac{3\sqrt{45}}{6}}{\frac{6}{6} + \frac{4\sqrt{45}}{6}}$

$= \frac{8-3\sqrt{45}}{\cancel{6}} \times \frac{\cancel{6}}{6+4\sqrt{45}}$

$= \frac{8-3\sqrt{45}}{6+4\sqrt{45}} = \boxed{\frac{8-9\sqrt{5}}{6+12\sqrt{5}}}$