

Unit 3 Test Review

Name: Answer Key

1. Factor the following expressions:

$$\begin{aligned} \text{a) } 8b^3 - 27 &= (2b-3)(4b^2 + 6b + 9) \\ &= (2b-3)\left(2b + \frac{3 + 3i\sqrt{3}}{2}\right)\left(2b + \frac{3 - 3i\sqrt{3}}{2}\right) \end{aligned}$$

$$\text{c) } 4c^2 + 12c + 9 = (2c + 3)^2$$

$$\text{d) } 3d^2 - 16d + 5 = (3d-1)(d-5)$$

2. Find the **exact** roots of the following polynomial equations:

$$\begin{aligned} \text{a) } 3x^4 - 7x^3 - 3x^2 - 7x - 6 &= 0 \\ x &= -2/3, 3, i, -i \end{aligned}$$

$$\begin{aligned} \text{b) } x^4 - 2x^3 - 14x^2 + 12x + 48 &= 0 \\ x &= -2, 4, \sqrt{6}, -\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{c) } 0 &= x^4 + 3x^2 - 4 \\ x &= -1, 1, 2i, -2i \end{aligned}$$

3. Is $(x+3)$ a factor of $f(x) = -2x^4 + 3x^3 - 4x^2 + x - 3$? How do you know?

$$f(-3) = -285 \neq 0 \quad \therefore (x+3) \text{ is } \underline{\text{NOT}} \text{ a factor by the Factor Theorem.}$$

4. Show whether -4 is a zero of $g(x) = x^3 - x^2 - 14x + 24$.

$$f(-4) = 0 \text{ therefore } -4 \text{ is a zero}$$

OR

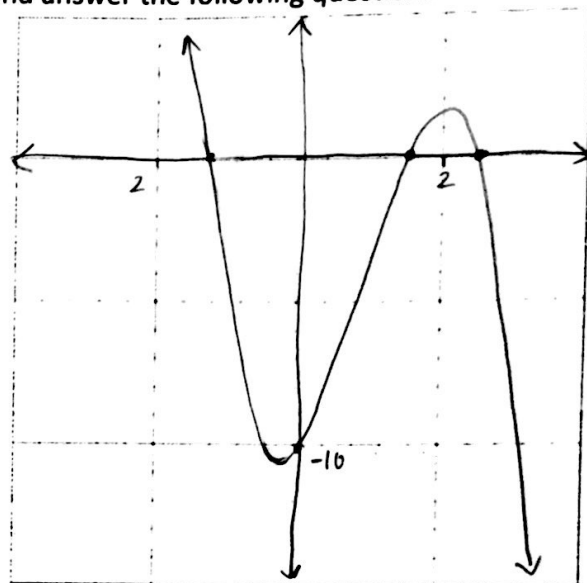
Remainder Theorem gives you a remainder of 0 for -4 therefore -4 is a zero

OR

Graph it on the graphing calculator...It crosses the x-axis at -4 therefore -4 is a zero.

5. Graph the function $p(x) = -2x^3 + 5x^2 + 4x - 10$, and answer the following questions:

- Relative maximum: $(2, 2)$
- Relative minimum: $(-0.333, -10.704)$
- Increasing interval: $(-0.333, 2)$
- Decreasing interval: $(-\infty, -0.333)$
 $(2, \infty)$
- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- End Behavior: $x \rightarrow \infty, y \rightarrow -\infty$
 $x \rightarrow -\infty, y \rightarrow \infty$
- Zeros: $-1.414, 1.414, 2.5$



6. The number of eggs, $f(x)$, in a female moth is a function of her abdominal width, x , in millimeters, modeled by $f(x) = 14x^3 - 17x^2 - 16x + 34$. What is the abdominal width when there are 211 eggs?

$$x = 3 \text{ mm} \left[\text{from the table of values for } f(x) \right]$$

7. The total number of books sold from 1995 to 2005 at Flyleaf Books can be modeled by the function $f(x) = 4x^3 + 14x^2 + 200x + 1560$ and the number of kinds of books in Flyleaf Books from 1995 to 2005 can be modeled by $g(x) = 2x + 12$, where x is the number of years since 1995. Using division, find the average number of each kind of book that Flyleaf Books sold.

$$\text{Average number of each kind of book} = 2x^2 - 5x + 130$$

8. Find the factored form of a polynomial that has a root at $\sqrt{5}$, a double root at -3 , and a root at $2i$. Assume the scale factor is 2.

$$f(x) = 2(x - \sqrt{5})(x + \sqrt{5})(x + 3)^2(x - 2i)(x + 2i)$$

cubic function

9. A polynomial has roots at -5 and $2/3$, and goes through the point (4, -30). Write the equation in factored form.

$$f(x) = (x+5)(x-2/3)(x+c) = (x+5)(x-2/3)(x-5)$$

use (4, -30) to find c

10. A cubic function has a root at $(3-2i)$ and goes through the point (3, 16). Write the equation in factored and general form.

$$f(x) = (x-(3-2i))(x-(3+2i))(x+c)$$

use (3, 16) to find c

$$= (x-(3-2i))(x-(3+2i))(x+1)$$

11. ~~DIAMONDS~~ The weight of an ideal round-cut diamond can be modeled by

~~$$w = 0.0071d^3 - 0.090d^2 + 0.48d$$~~

~~where w is the diamond's weight (in carats) and d is~~

12. **BUSINESS** For the 12 years that a grocery store has been open, its annual revenue R (in millions of dollars) can be modeled by the function

$$R = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13,650)$$

where t is the number of years since the store opened. In which year(s) was the revenue \$1.5 million?

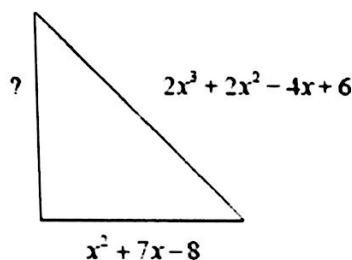
$t = 3$ years since the store opened.

[from the table of $R(t)$]

13. Given the perimeter, P , of a triangle, find the length of the missing side, in terms of x .

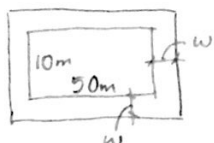
$$P = 2x^3 + 4x^2 + 6x + 3$$

$$? = x^2 + 3x + 5$$



14. A

concrete walkway of uniform width surrounds a rectangular swimming pool that is 10 m wide and 50 m long. Find the width of the walk if its area is 864 m^2 .



$$\begin{aligned} \text{Area} &= (50 + 2w)(10 + 2w) - (10)(50) \\ 864 &= 500 + 120w + 4w^2 - 500 \\ w &= 6 \text{ m} \end{aligned}$$

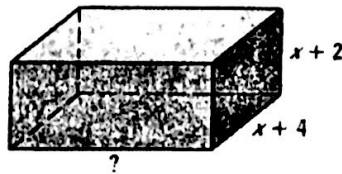
15. Given the volume, V , of a box, find the length of the missing side, in terms of x .

NEXT PAGE



$$V = 2x^3 + 17x^2 + 46x + 40$$

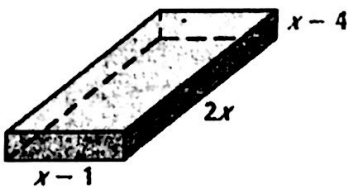
$$? = 2x + 5$$



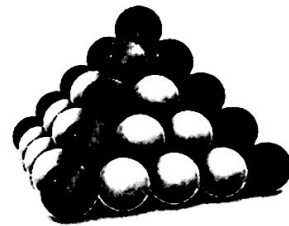
16. Given the volume, V , of a box, solve for x .

Volume = 40

$$x = 5 \quad \left[\text{from the table of } V = (x-1)(2x)(x-4) \right]$$



17. A pyramid can be formed using equal-size balls. For example, 3 balls can be arranged in a triangle, then a fourth ball placed in the middle on top of them. The function $p(n) = \frac{1}{6}n(n+1)(n+2)$ gives the number of balls in a pyramid, where n is the number of balls on each side of the bottom layer. (For the pyramid described above, $n = 2$. For the pyramid in the picture, $n = 5$.)



- a. Evaluate $p(2)$, $p(3)$, and $p(4)$. Sketch a picture of the pyramid that goes with each of these values. Check that your function values agree with your pyramid pictures.

$$p(2) = 4$$

$$p(3) = 10$$

$$p(4) = 20$$

- b. If you had 1000 balls available and you wanted to make the largest possible pyramid using them, what would be the size of the bottom triangle, and how many balls would you use to make the pyramid? How many balls would be left over?

$$p(17) = 969 \quad \left[\text{from the table of } p(n) \right]$$

$$n = 17$$

$$1000 - 969 = 31 \text{ left over}$$