

SAS 10 – Expected Value

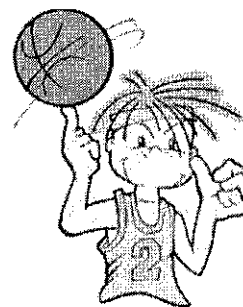
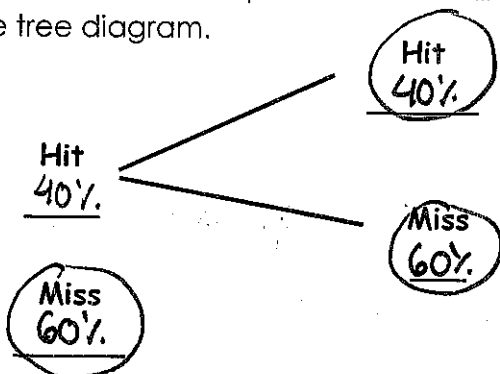
Part 1: Candace's Deal

Candace Parker gets \$15 a week for allowance. She also loves to play basketball. Candace wants to convince her father to try something new with her allowance based on her free-throw shooting abilities, hoping that it will increase what she receives each week. Candace suggests that instead of getting \$15, she attempt two free-throws each week for her allowance.

- If she misses the first free-throw, she gets only \$5 and no chance for a second shot.
- If she makes the first free-throw, she gets \$15 and a chance to make a second free-throw for an additional \$10.

Candace can currently make free-throws 40% of the time.

- To help Candace's father decide whether to use the new allowance plan, find the probability of Candace making 0 free-throws, 1 free-throw, and 2 free-throws. Justify your reasoning with a tree diagram. Because she has different probabilities it will be a weighted tree diagram. Fill in the probabilities on the tree diagram.



Probability of \$5 (0 baskets)	Probability of \$15 (1 baskets)	Probability of \$25 (2 baskets)
60%	$(.40)(.60) = .24$ 24%	$(.40)(.40) = .16$ 16%

- How much money do you expect Candace to make any given week?

$$\$5(.60) + \$15(.24) + \$25(.16) = \$10.60$$

- How much do you expect Candace to make in a year? (52 weeks in a year)

$$\$10.60 \times 52 = \$551.20$$

- How much allowance was Candace making before this deal?

was making \$15 a week so \$780 per year

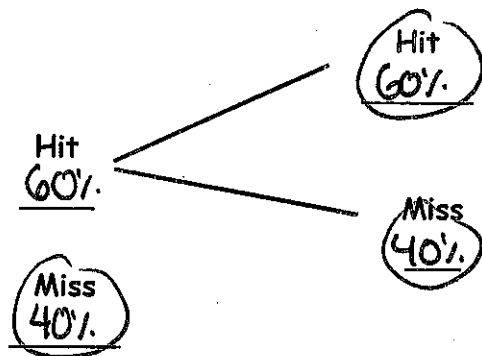
- Should Candace's father accept the deal? Justify your reasoning

HER FATHER SHOULD BECAUSE HE ENDS TO MAKE AN EXTRA \$228.80 EACH YEAR.

Part 2: Candace's Practice Paid Off

Candace practiced shooting free-throws for an entire year. She can now make a free-throw 60% of the time. Candace offers her father the same deal.

6. What is the probability of Candace making 0 free-throws? 1 free-throw? 2 free-throws?



Probability of \$5 (0 baskets)	Probability of \$15 (1 baskets)	Probability of \$25 (2 baskets)
40%	$(.60)(.40) = .24$	$(.60)(.60) = .36$
40%	24%	36%

7. Now that she is better at free throws, how much money do you expect Candace to receive in a year?

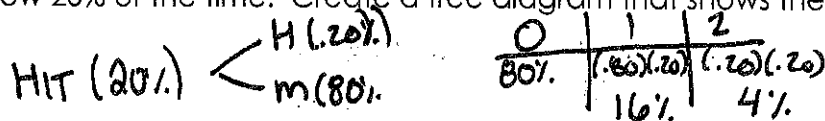
$$\$5(.40) + \$15(.24) + \$25(.36) = \$14.60$$

8. Should her father take the deal now? Justify your reasoning.

YES HE STILL EXPECTS TO ~~LOSE~~ SAVE \$1.40 A WEEK OR \$2080 A YEAR.

Part 3: Candace's Friend Sheryl

Candace's friend, Sheryl Swoops, wants her father to offer her the same deal. Sheryl can make a free-throw 20% of the time. Create a tree diagram that shows the possible outcomes



$$\$5(.80) + \$15(.16) + \$25(.04) =$$

9. How much do you expect Sheryl to make in a week?

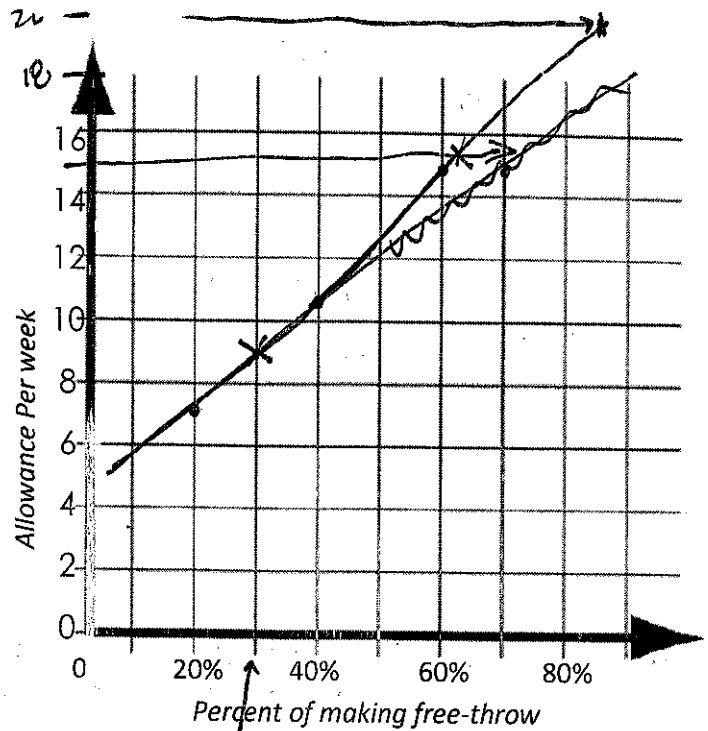
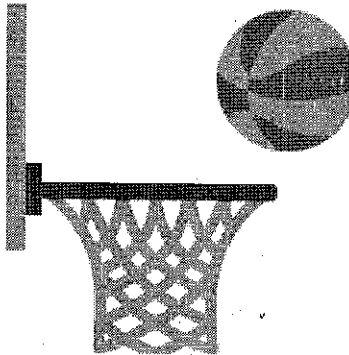
$$\$7.40$$

10. How much do you expect her to make in a year?

$$\$384.80$$

You have now looked at Candace's situation (with a 40% & 60% shooting percentages) and Sheryl's (with a 20% shooting percentage) to determine what percent of the time they are each likely to receive \$5, \$15, and \$25 in allowance. Fill in the table below.

Free-Throw % (x)	Average Allowance Per Week (y)
Sheryl (20%)	\$7.40
Candace (40%)	\$10.60
Candace (60%)	\$14.60



11. Sheryl's ability to make a free-throw improved to 30%. Using your graph, what average weekly allowance should she expect?

PROBABLY ABOUT \$9

12. If Candace wants to earn an average weekly allowance of \$15, what percentage of free-throws must she make? Use the graph to estimate a number then justify your answer with expected value.

A LITTLE OVER 60%. TRY 65%.

SO 65% IS A LITTLE OVER \$15 A WEEK

	0	1	2
H 65%			
m 35%			
	\$5	(.65)(.35)	(.65)(.65)
		22.75%	42.25%

m 35%

$$\$5(.35) + \$15(.2275) + \$25(.4225) = \$15.73$$

13. Candace's father figured out that he had been giving Candace \$20 per week for her allowance. What percentage of free-throws is Candace making? Justify your answer using expected values. (your answer should round to \$20 as a whole number)

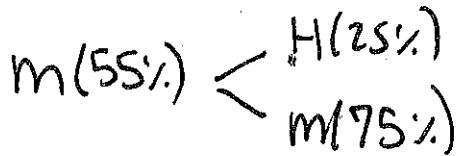
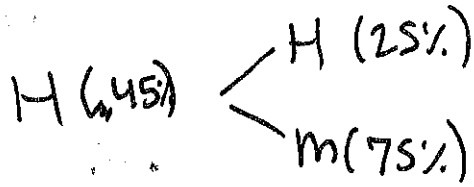
A LITTLE OVER 80%. ON THE GRAPH TRY 82%.

	0	1	2
		(.82)(.18)	(.82)(.82)
	18%	14.76%	67.24%

$$\$5(.18) + \$15(.1476) + \$25(.6724) = \$19.92$$

WOULD ROUND UP TO \$20

14. Harry made a carnival game where you have to throw a ball into a basket. The probability of making the first shot is 45% and the probability of the second shot is 25%. Everybody takes 2 shots. Draw a tree diagram that shows the different outcomes



Probability of missing both shots	Probability of Miss first shot/ Make second shot	Probability of Make first shot/miss second shot	Probability of making both shots
$(.55)(.75)$ 41.25%	$(.55)(.25)$ 13.75%	$(.45)(.75)$ 33.75%	$(.45)(.25)$ 11.25%

Harry is planning on giving away the following prizes.

- No shots - no money
- Make First Shot only - \$1
- Make Second shot only - \$2
- Make both shots - \$7

15. Janet plays the game. What can she expect to win if she plays the game?

$$\$0(.4125) + \$2(.1375) + \$1(.3375) + \$7(.1125) = \$1.40$$

16. If it costs \$1 to play the game would Janet have a positive or a negative expected value? Should she play the game? Why or why not?

JANET WOULD BE POSITIVE \$.40. SHE SHOULD PLAY THE GAME, SHE EXPECTS TO WIN MORE THAN SHE SPENDS!

17. What would you suggest Harry charge to play his game? Why?

\$2 SO THAT WAY HARRY EXPECTS TO COVER HIS EXPENSES. (REALLY ANYTHING OVER \$1.40 WILL HAVE HIM IN EXPECTED PROFIT)