

## HW 2.3 – Product Rule and Quotient Rule and Higher Order Derivatives

Find and completely simplify (in factored form) the derivative of each function using the Product Rule.

1.)  $f(x) = (2x-3)(1-5x)$

$$f(x) = -10x^2 + 17x - 3$$

$$f'(x) = -20x + 17$$

2.)  $y = (3x-4)(x^3+5)$

3.)  $y = (x^2-4x-6)(x^3-5x^2-3x)$

$$y' = (x^2-4x-6)(3x^2-10x-3) + (x^3-5x^2-3x)(2x-4)$$

4.)  $f(t) = \sqrt{t}(1-t^2)$

$$f(t) = t^{1/2} - t^{5/2}$$

$$f'(t) = \frac{1}{2}t^{-1/2} - \frac{5}{2}t^{3/2}$$

$$= \frac{1}{2t^{1/2}} - \frac{5}{2}t^{3/2}$$

5.)  $f(t) = t^2 \sin t$

$$f'(t) = t^2 \cos t + 2t \sin t$$

6.)  $f(x) = 2x \sin x + x^2 e^x$

$$f'(x) = 2x \cos x + 2 \sin x + x^2 e^x + 2x e^x$$

Find and completely simplify (in factored form) the derivative of each function using the Quotient Rule.

7.)  $y = \frac{3x-2}{2x+3}$

$$y' = \frac{(2x+3)3 - (3x-2)(2)}{(2x+3)^2}$$

$$= \frac{6x+9 - 6x+4}{(2x+3)^2}$$

$$= \frac{13}{(2x+3)^2}$$

8.)  $f(x) = \frac{x^2-4x-2}{x^2-1}$

8.)  $f(x) = \frac{x^2}{2\sqrt{x+1}}$

9.)  $y = \frac{\sin x}{e^x}$

$$y' = \frac{e^x \cos x - e^x \sin x}{(e^x)^2}$$

$$= \frac{\cos x - \sin x}{e^x}$$

11.)  $f(x) = \frac{3(1-\sin x)}{2\cos x} = \frac{3-3\sin x}{2\cos x}$

$$= \frac{3}{2\cos x} - \frac{3\sin x}{2\cos x}$$

$$= \frac{3}{2} \sec x - \frac{3}{2} \tan x$$

$$f'(x) = \frac{3}{2} \sec x \tan x - \frac{3}{2} \sec^2 x$$

12.)  $f(x) = \frac{2-\frac{1}{x}}{x-3}$

13.)  $y = \frac{x^2 + k^2}{x^2 - k^2}$ ,  $k$  is a constant

14.)  $y = \frac{x^2 - k^2}{x^2 + k^2}$ ,  $k$  is a constant

15.)  $y = 2x \sin x + x^2 e^x$

$$y' = 2x \cos x + 2 \sin x + x^2 e^x + 2x e^x$$

16.)  $y = \left(\frac{x-3}{x+4}\right)(3x-2)$

**Bonus!**

$$y' = \left(\frac{x-3}{x+4}\right)'(3x-2) + (3x-2)\left(\frac{x+4-(x-3)}{(x+4)^2}\right)$$

$$= \frac{3x-9}{x+4} + (3x-2)\left(\frac{7}{(x+4)^2}\right)$$

$$= \frac{3x-9}{x+4} + \frac{21x-14}{(x+4)^2} = \frac{(x+4)(3x-9)}{(x+4)^2} + \frac{21x-14}{(x+4)^2}$$

Find an equation of the tangent line to the graph of  $f$  at the indicated point and then use your calculator to confirm the results.

17.)  $f(x) = \frac{x^2}{x-1}$  at  $(2, 4)$

$$f'(x) = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

$f'(2) = 0$       $y = 4$       $y - 4 = 0(x - 2)$

18.)  $f(x) = (x-2)(x^2 - 3x - 1)$  at  $(-1, -9)$

$$\frac{3x^2 + 3x - 36 + 21x - 14}{(x+4)^2}$$

$$\frac{3x^2 + 24x - 50}{(x+4)^2}$$

19.)  $f(x) = \frac{x^2 - 4x + 2}{2x - 1}$  at  $\left(2, -\frac{2}{3}\right)$

20.)  $f(x) = \left(\frac{x+3}{x+1}\right)(4x+1)$  at  $\left(-\frac{1}{2}, -5\right)$

Determine the point(s) at which the graph of the following function as a horizontal tangent.

21.)  $f(x) = \frac{x^2}{x^2 - 4}$

22.)  $f(x) = \frac{4x}{x^2 + 4}$

$$f'(x) = \frac{(x^2 + 4)4 - 4x(2x)}{(x^2 + 4)^2}$$

$$= \frac{4x^2 + 16 - 8x^2}{(x^2 + 4)^2}$$

$$= \frac{-4x^2 + 16}{(x^2 + 4)^2} = 0$$

$-4x^2 + 16 = 0$   
 $x^2 = 4 \rightarrow x = 2, -2$

Use the chart to the right to find  $h'(4)$  for Problems #23-28.

| $f(4)$ | $f'(4)$ | $g(4)$ | $g'(4)$ |
|--------|---------|--------|---------|
| -8     | 3       | $3\pi$ | 4       |

$(2, 1)$   
 $(-2, -1)$

23.)  $h(x) = 5f(x) - \frac{2}{3}g(x)$

24.)  $h(x) = 3 + 8f(x)$

25.)  $h(x) = f(x)g(x)$

26.)  $h(x) = \frac{f(x)}{g(x)}$

27.)  $h(x) = \frac{g(x)}{f(x)}$

28.)  $h(x) = \frac{f(x) + 2}{-3g(x)}$

For each problem, find the indicated derivative with respect to  $x$ .

29.)  $f(x) = 3 \csc x$   
 $f'(x) = -3 \csc x \cot x$

30.)  $f(x) = \frac{-\tan x}{3} = -\frac{1}{3} \tan x$   
 $f'(x) = -\frac{1}{3} \sec^2 x$

For each of the following, find  ~~$f''(x)$~~ .  $f'(x)$

31.)  $f(x) = \frac{x^3 - 3x^2 - 4x - 1}{2x}$   
 $f(x) = \frac{1}{2}x^2 - \frac{3}{2}x - 2 - \frac{1}{2}x^{-1}$   
 $f'(x) = x - \frac{3}{2} + \frac{1}{2x^2}$   
 $\frac{x^3}{2x} - \frac{3x^2}{2x} - \frac{4x}{2x} - \frac{1}{2x}$

32.)  $f(x) = \frac{x}{x-4}$

33.)  $f(x) = \sqrt{x} - 4\sqrt[3]{x} + \frac{6}{5\sqrt[4]{x}}$   
 $f(x) = x^{1/2} - 4x^{1/3} + \frac{6}{5}x^{-1/4}$   
 $f'(x) = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-2/3} - \frac{3}{10}x^{-5/4}$   
 $= \frac{1}{2\sqrt{x}} - \frac{4}{3\sqrt[3]{x^2}} + \frac{3}{10\sqrt[4]{x^5}}$

Find an equation of the line that is tangent to  $f(x) = x^2 - 6x + 7$  and...

34.) ...parallel to the line  $y = 2x + 4$

35.) ...perpendicular to the line  $y = 2x + 4$

For each problem, find the indicated derivative with respect to  $x$ .

36.)  $y = -x^2$  Find  $\frac{d^2y}{dx^2}$

37.)  $f(x) = 4x^3$  Find  $f''$

|   |   |   |
|---|---|---|
| <p>38.) <math>y = -x^2 + 2\sqrt[5]{x^2}</math> Find <math>\frac{d^3 y}{dx^3}</math></p> | <p>39.) <math>y = -2x^3 - 4x^{-3}</math> Find <math>\frac{d^3 y}{dx^3}</math></p> |   |
| <p>40.) <math>y = 99x^{99}</math> Find <math>\frac{d^{100} y}{dx^{100}}</math></p>      | <p>41.) <math>f(x) = x^{99}</math> Find <math>f^{(99)}</math></p>                 | <p>42.) <math>f(x) = \sin x</math> Find <math>f^{(43)}</math></p> |

For each problem, find the indicated derivative with respect to  $x$ .

|  |  |
|--|--|
| <p>43.) <math>f(x) =  3x + 7 </math></p> | <p>44.) <math>f(x) =  x^2 - 2x - 8 </math></p> |
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