Key

Continuity HW **Day 3**

1.) **State whether or not the function is continuous at the value where the rule for the function changes. If continuous, state the three criteria of continuity. If not, state if the discontinuity is removable or non-removable. If it is non-removable, identify the type.**

a.) $f(x)=\begin{cases}8-x^2, & x<2\\ 6-x, & x\ge 2\end{cases}$ $\lim_{x\to 2^-} 8-x^2 = 4$ $f(2)=4$ $\lim_{x\to 2^+} 6-x = 4$ Cont @ $x=4$	b.) $f(x)=\begin{cases}4-x^2, & x<1\\ 1+x, & x\ge 1\end{cases}$ $\lim_{x\to 1} f(x) = \text{DNE}$ $\lim_{x\to 1^-} 4-x^2=3$ $\lim_{x\to 1^+} 1+x=2$ Discont @ $x=1$ non-rem. jump
c.) $f(x)=\begin{cases}2^x, & x<3\\ 10-x, & x\ge 3\end{cases}$ $\lim_{x\to 3^-} 2^x = 8$ $\lim_{x\to 3^+} 10-x=7$ $\lim_{x\to 3} f(x) =$ DNE Discont @ $x=3$ non-rem. jump	d.) $f(x)=\begin{cases}2^{-x}, & x<-1\\ x+3, & x\ge -1\end{cases}$ $f(-1)=2$ $\lim_{x\to -1^-} 2^{-x}=2$ $\lim_{x\to -1^+} x+3=2$ Cont @ $x=-1$
e.) $f(x)=\begin{cases}\dfrac{1}{x-2}, & x<2\\ 3, & x=2\\ x+1, & x>2\end{cases}$ $\lim_{x\to 2^-} \frac{1}{x-2} = \frac{1}{0^-} = -\infty$ $\lim_{x\to 2^+} x+1 = 3$ Discont. @ $x=2$ non. rem. inf.	f.) $f(x)=\begin{cases}\dfrac{x^3-x}{x^2-x}, & x\ne 0,\ x\ne 1\\ 3, & x=0\\ 2, & x=1\end{cases}$ $\frac{x(\cancel{x-1})(x+1)}{\cancel{x}(\cancel{x-1})}$ $\lim_{x\to 0} \frac{x^3-x}{x^2-x} = x+1 = 1$ $f(0)=3$ Discont @ $x=0$ removable $\lim_{x\to 1} \frac{x^3-x}{x^2-x} = x+1=2$ $f(1)=2$ Cont @ $x=1$

2.) **Find the value of the constant *a* which makes the function continuous. Be sure to justify your solution using proper notation.**

a.) $f(x)=\begin{cases}x^2, & x>2\\ a-x, & x\le 2\end{cases}$ $\lim_{x\to 2^-} x^2=4$ $\lim_{x\to 2^+} a-x = a-2$ $4=a-2$ $a=6$	b.) $f(x)=\begin{cases}9-x^2, & x>2\\ ax, & x\le 2\end{cases}$ $\lim_{x\to 2^-} 9-x^2=5$ $\lim_{x\to 2^+} ax = 2a$ $2a=5$ $a=5/2$

c.) $f(x)=\begin{cases} ax+5, & x\le -1 \\ ax^2, & x>-1 \end{cases}$

$\lim_{x\to -1^-} -a+5$

$\lim_{x\to -1^+} a$

$a=-a+5$

$2a=5$

$a=5/2$

d.) $f(x)=\begin{cases} a^2-x^2, & x<2 \\ 1.5ax, & x\ge 2 \end{cases}$

$\lim_{x\to 2^-} a^2-x^2=a^2-4$

$\lim_{x\to 2^+} 1.5ax=3a$

$a^2-4=3a$

$a^2-3a-4=0$

$(a-4)(a+1)=0$

$a=4,-1$

3.) Let a and b stand for constants and let $f(x)=\begin{cases} b-x, & x<1 \\ a(x-2)^2, & x\ge 1 \end{cases}$.

a.) Find an equation relating a and b if f is to be continuous at $x=1$.

$\lim_{x\to 1^-} b-x=b-1$

$\lim_{x\to 1^+} a(x-2)^2=a$

$a=b-1$

b.) Find b if $a=-1$. Graph and show that the function is continuous.

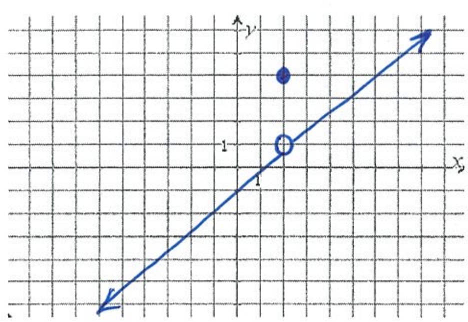
$-1=b-1$

$b=0$

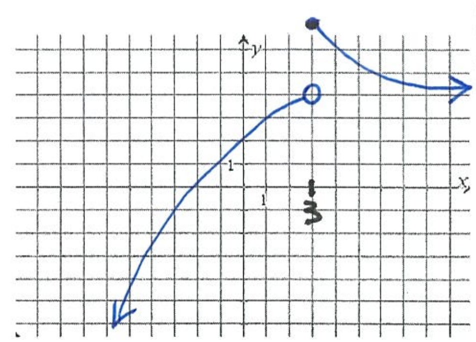
$f(x)=\begin{cases} -x, & x<1 \\ -(x-2)^2, & x\ge 1 \end{cases}$

4.) Sketch a function having the following attributes.

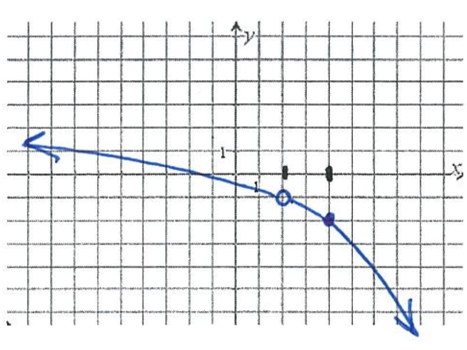
a.) has a value of $f(2)$, a limit as x approaches 2, but is not continuous at $x=2$.



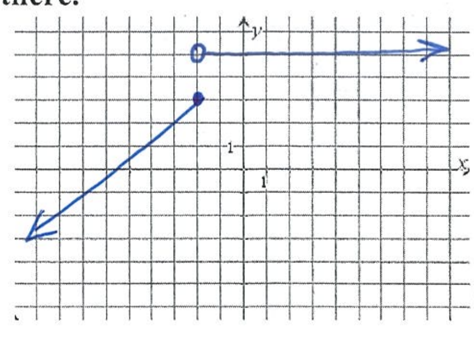
b.) has a step discontinuity at $x=3$ where $f(3)=7$.



c.) $\lim_{x\to 4} f(x)=-2$ but the function is not continuous at $x=2$.



d.) the value of $f(-2)=3$ but there is no limit of $f(x)$ as x approaches -2 and no vertical asymptotes there.



Calculus Name________________________________

Continuity WS

Date____________________ Period____

Determine if each function is continuous. If the function is not continuous, classify the discontinuity as removable or non-removable and if non-removable identify the type (inifnite, jump, oscillating).

1) $f(x)=\dfrac{\cancel{x+3}}{x^2+5x+6}$

$(\cancel{x+3})(x+2)$

Discont at $x=-3$ rem. hole

$x=-2$ non-rem inf.

2) $f(x)=\sin\dfrac{1}{x+\pi}$

Look at graph

Discont @ $x=-\pi$ non-rem. oscillating

3) $f(x)=\begin{cases}1, & x\neq 2\\ 4, & x=2\end{cases}$

$\lim_{x\to 2} f(x)=4$

$f(2)=1$

$\lim_{x\to 2} f(x)\neq f(2)$

Discont @ $x=2$ rem. hole

4) $f(x)=\cos\dfrac{1}{x-\pi}$

Discont @ $x=\pi$ non-rem. oscillating

5) $f(x)=-\dfrac{x+3}{x^2+2x-3}$

$\dfrac{-(\cancel{x+3})}{(\cancel{x+3})(x-1)}$

Discont @ $x=-3$ rem. hole

@ $x=1$ non-rem. inf.

6) $f(x)=\dfrac{x}{x^2+2x+1}$

$(x+1)(x+1)$

Discont @ $x=-1$ non-rem. inf.

7) $f(x)=-\dfrac{x-3}{x^2-5x+6}$

$\dfrac{-(\cancel{x-3})}{(\cancel{x-3})(x-2)}$

Discont @ $x=3$ rem., hole

@ $x=2$ non-rem, inf.

8) $f(x)=\dfrac{x^2+2x}{x+2}$

$\dfrac{x(\cancel{x+2})}{\cancel{x+2}}$

Discont. @ $x=-2$ rem. hole

9) $f(x)=-\dfrac{x^2+x-2}{x-1}$

$\dfrac{-(x+2)(\cancel{x-1})}{\cancel{x-1}}$

Discont. @ $x=1$ rem., hole

10) $f(x)=-\dfrac{x-1}{x^2+2x-3}$

$\dfrac{-(\cancel{x-1})}{(x+3)(\cancel{x-1})}$

Discont @ $x=1$, rem, hole

@ $x=3$, non-rem, inf.