Calculus Honors
Continuity Assignment

Name Key

State whether or not each of the following functions is continuous. If not, state where the discontinuity occurs and whether or not it is removable. Is the discontinuity is an asymptote, a hole, or a jump? If it is an asymptote, what is its equation?

1) $f(x) = \dfrac{x}{x^2+1}$ $\neq 0$

always cont.

2) $f(x) = \dfrac{x}{2x^2-x-1} = \dfrac{x}{(2x+1)(x-1)}$

Discont @ $x = -\frac{1}{2}$, $x = 1$ non-rem. inf.

3) $f(x) = \dfrac{2x+3}{x^2-x-6} = \dfrac{2x+3}{(x-3)(x+2)}$

Discont. @ $x=3$, $x=-2$ non-rem. inf.

4) $f(x) = \dfrac{x-4}{x^2-16} = \dfrac{\cancel{x-4}}{(\cancel{x-4})(x+4)}$

Discont. @ $x=4$ rem. → hole

$x=-4$ non-rem. inf.

5) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 8 & \text{if } x = 3 \end{cases}$

$\lim\limits_{x\to 3} \dfrac{(\cancel{x-3})(x+3)}{\cancel{x-3}} = 6$

Discont. @ $x=3$ remov., hole

6) $f(x) = \begin{cases} 2x-3 & \text{if } x \le 2 \\ x^2 & \text{if } x > 2 \end{cases}$

$\lim\limits_{x\to 2^-} 2x-3 = 1$

$\lim\limits_{x\to 2^+} x^2 = 4$

Discont @ $x=2$ non-rem. jump.

7) $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ 1-x & \text{if } x \ge 1 \end{cases}$

$\underbrace{\lim\limits_{x\to -1^-} x^3 = -1 \qquad \lim\limits_{x\to -1^+} x = -1}_{\text{Cont. @ } x=-1}$

$f(-1) = -1$

$\underbrace{\lim\limits_{x\to 1^-} x = 1 \qquad \lim\limits_{x\to 1^+} 1-x = 0}_{\text{Discont @ } x=1}$

non-rem. jump

8) $f(x) = \dfrac{x}{|x|-3}$

$|x|-3=0$

$|x|=3$ Disc @ $x = 3, -3$

non-rem. inf.

Find the value of "a" and/or "b" for which the function is continuous.

9)
$$f(x) = \begin{cases} 7x - 2 & \text{if } x \le 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

$$\lim_{x \to 1^-} 7x - 2 = 5$$

$$\lim_{x \to 1^+} ax^2 = a$$

$$\boxed{a = 5}$$

10)
$$f(x) = \begin{cases} ax^2 & \text{if } x \le 2 \\ 2x + a & \text{if } x > 2 \end{cases}$$

$$\lim_{x \to 2^-} ax^2 = 4a$$

$$\lim_{x \to 2^+} 2x + a = a + 4$$

$$4a = a + 4$$

$$3a = 4$$

$$a = \frac{4}{3}$$

11)
$$f(x) = \begin{cases} x + 1 & \text{if } x < 1 \\ ax + b & \text{if } 1 \le x < 2 \\ 3x & \text{if } x \ge 2 \end{cases}$$

$$\lim_{x \to 1^-} x + 1 = 2$$

$$\lim_{x \to 1^+} ax + b = a + b$$

$a + b = 2$

$$\lim_{x \to 2^-} ax + b = 2a + b$$

$$\lim_{x \to 2^+} 3x = 6$$

$2a + b = 6$

$-a + \cancel{-b} = -2$

$2a + \cancel{b} = 6$

$a = 4$

$4 + b = 2$

$b = -2$

Are the following functions continuous at all points in the natural domain? If the function is not continuous, does it have a removable discontinuity? If it has a removable discontinuity, create a continuous function.

12) $f(x) = \dfrac{x^2 - 16}{x + 4}$

$$\lim_{x \to -4} \frac{(\cancel{x+4})(x-4)}{\cancel{x+4}} = -8$$

removable discont @ $x = -4$

$$f(x) = \begin{cases} \dfrac{x^2-16}{x+4}, & x \neq -4 \\ -8, & x = -4 \end{cases}$$

13) $f(x) = \dfrac{2x^2 - x - 1}{x - 1}$

$$\lim_{x \to 1} \frac{(2x+1)(\cancel{x-1})}{\cancel{x-1}} = 3$$

removable discont @ $x = 1$

$$f(x) = \begin{cases} \dfrac{2x^2-x-1}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$$

14) $f(x) = \dfrac{9x^2 - 4}{3x + 2}$

$$\lim_{x \to -2/3} \frac{(3x-2)(\cancel{3x+2})}{\cancel{3x+2}} = -4$$

$$f(x) = \begin{cases} \dfrac{9x^2-4}{3x+2}, & x \neq -2/3 \\ -4, & x = -2/3 \end{cases}$$

15) $g(t) = \dfrac{\sin t}{t}$

$t \neq 0$

$g(0)$ is und.

Discont @ $t = 0$

non-removabl

Special EX.

Trick Problem

ex $y = \dfrac{2x^2 - 5x + 2}{(x^2 - 4x + 4)(2x - 1)}$

$$y = \frac{(\cancel{2x-1})(\cancel{x-2})}{(\cancel{2x-1})(x-2)^{\cancel{2}}}$$

Discont @ $x = 1/2$, rem.

Discont @ $x = 2$, non-rem inf.