Key

Chain Rule - Transcendental Functions

Find the derivative of each of the following functions.

1.) $f(x) = 3\sin 2x$

$f'(x) = 3\cos 2x \cdot 2$

$= 6\cos 2x$

2.) $f(x) = 6\cos(x^2)$

$f'(x) = -6\sin x^2 \cdot 2x$

$= -12x\sin x^2$

3.) $f(x) = \tan\sqrt{x}$ $= \tan(x^{1/2})$

$f'(x) = \sec^2(x^{1/2}) \cdot \frac{1}{2}x^{-1/2}$

$= \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$

4.) $f(x) = \sin(\pi x)^4$

$f'(x) = \cos(\pi x)^4 \cdot 4(\pi x)^3 \cdot \pi$

$= 4\pi(\pi x)^3 \cos(\pi x)^4$

or

$f(x) = \sin(\pi^4 x^4)$

$f'(x) = \cos(\pi^4 x^4) \cdot 4\pi^4 x^3$

$= 4\pi^4 x^3 \cos(\pi^4 x^4)$

5.) $f(x) = \sec^2(x)$

$f'(x) = 2\sec x \sec x \tan x$

$= 2\sec^2 x \tan x$

6.) $f(x) = \sin^2(3x)$

$f'(x) = 2\sin(3x)\cos(3x) \cdot 3$

$= 6\sin 3x \cos 3x$

7.) $f(x) = \cos^4(x^4)$

$f'(x) = 4\cos^3(x^4) \cdot (-\sin x^4) \cdot 4x^3$

$= -16x^3 \cos^3 x^4 \sin x^4$

8.) $f(x) = \cot^3\left(\frac{x}{3}\right)$

$3\cot^2\left(\frac{x}{3}\right)\left(-\csc^2\left(\frac{x}{3}\right)\right) \cdot \frac{1}{3}$

$-\cot^2\left(\frac{x}{3}\right)\csc^2\left(\frac{x}{3}\right)$

9.) $y = e^{4x}$

$$y' = 4e^{4x}$$

10.) $y = 16e^{-2x}$

$$y' = 16(-2e^{-2x})$$
$$= -32e^{-2x}$$

11.) $y = (e^x - 2x - 1)^3$

$$y' = 3(e^x - 2x - 1)^2(e^x - 2)$$

12.) $y = \sqrt{e^{4x} - 4x} = (e^{4x} - 4x)^{1/2}$

$$y' = \frac{1}{2}(e^{4x} - 4x)^{-1/2}(4e^{4x} - 4)$$
$$= \frac{4e^{4x} - 4}{2\sqrt{e^{4x} - 4x}}$$
$$= \frac{2e^{4x} - 2}{\sqrt{e^{4x} - 4x}}$$

13.) $y = 10^{x^2 - \sin x}$

$$y' = \ln 10\left(10^{x^2 - \sin x}\right)(2x - \cos x)$$

14.) $y = 3^{5x}$

$$y' = \ln 3(3^{5x})(5)$$
$$= 5\ln 3(3^{5x})$$

15.) $y = \ln(x^6)$

$$y' = \frac{1}{x^6} \cdot 6x^5 = \frac{6}{x}$$

OR

$$y = 6\ln x$$
$$= \frac{6}{x}$$

16.) $y = (\ln x)^7$

$$y' = 7(\ln x)^6 \cdot \frac{1}{x}$$
$$= \frac{7(\ln x)^6}{x}$$

17.) $y=\log_3(x^2+1)$

$$y'=\frac{1}{\ln 3}\cdot\frac{1}{x^2+1}\cdot 2x$$

$$=\frac{2x}{\ln 3(x^2+1)}$$

18.) $y=4\log 3x$

$$y'=4\cdot\frac{1}{\ln 10}\cdot\frac{1}{3x}\cdot 3$$

$$=\frac{12}{3x\ln 10}$$

19.) $y=\ln(\sin x)$

$$y'=\frac{1}{\sin x}\cdot\cos x$$

$$=\cot x$$

20.) $y=x\ln(\tan x)$

$$y'=x\left[\frac{1}{\tan x}\cdot\sec^2 x\right]+\ln(\tan x)$$

$$=\frac{x}{\sin x\cos x}+\ln(\tan x)$$

21.) $y=\ln\left[(3x^2-3x+2)(5x-1)\right]$

$$\ln(3x^2-3x+2)+\ln(5x-1)$$

$$\frac{1}{3x^2-3x+2}(6x-3)+\frac{1}{5x-1}\cdot 5$$

$$\frac{6x-3}{3x^2-3x+2}+\frac{5}{5x-1}$$

22.) $y=\ln\left(\dfrac{3x^2-3x+2}{5x-1}\right)$

$$\ln(3x^2-3x+2)-\ln(5x-1)$$

$$\frac{6x-3}{3x^2-3x+2}-\frac{5}{5x-1}$$

23.) $y=\ln\sqrt{x^2-4x-7}$

$$y=\ln(x^2-4x-7)^{1/2}$$

$$y=\frac{1}{2}\ln(x^2-4x-7)$$

$$y'=\frac{1}{2(x^2-4x-7)}(2x-4)$$

$$=\frac{x-2}{x^2-4x-7}$$

24.) $s(t)=\ln\left[x^3\cdot\sqrt[3]{\dfrac{3x-1}{3x+1}}\right]$

$$s(t)=3\ln x+\frac{1}{3}\ln(3x-1)-\frac{1}{3}\ln(3x+$$

$$s'(t)=\frac{3}{x}+\frac{1}{\cancel{3}}\cdot\frac{1}{3x-1}\cdot\cancel{3}-\frac{1}{\cancel{3}}\frac{1}{3x+1}\cdot\cancel{3}$$

$$=\frac{3}{x}+\frac{1}{3x-1}-\frac{1}{3x+1}$$