

Solve each of the following. Watch the bounded vs. unbounded.

1. $4 \cos x = 1 + 2 \cos x, [0, 2\pi)$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

2. $4 \tan^2 x - 1 = \tan^2 x$

$$3 \tan^2 x = 1$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{6} + \pi n$$

$$x = \frac{5\pi}{6} + \pi n \quad \left. \vphantom{x = \frac{\pi}{6} + \pi n} \right\} n \in \mathbb{Z}$$

3. $\sin^2 x + 2 \cos x = 2, [0, 2\pi)$

$$1 - \cos^2 x + 2 \cos x = 2$$

$$\cos^2 x - 2 \cos x + 1 = 0$$

$$(\cos x - 1)^2 = 0$$

$$\cos x = 1$$

$$x = 0$$

4. $4 \sin 2x \cos^2 2x = \sin 2x$

$$4 \sin 2x \cos^2 2x - \sin 2x = 0$$

$$\sin 2x (4 \cos^2 2x - 1) = 0$$

$$\sin 2x = 0$$

$$\cos^2 2x = \frac{1}{4}$$

$$\cos 2x = \pm \frac{1}{2}$$

$$2x = 0 + \pi n$$

$$= \pi n$$

$$2x = \frac{\pi}{3} + \pi n$$

$$2x = \frac{2\pi}{3} + \pi n$$

$$x = \frac{\pi}{2} n, n \in \mathbb{Z}$$

5. $\cos\left(x - \frac{\pi}{3}\right) - \cos\left(x + \frac{\pi}{3}\right) = 1, [0, 2\pi)$

$$\cos x \cos \frac{\pi}{3} + \sin x \sin \frac{\pi}{3} - (\cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}) = 1$$

$$2 \sin x \sin \frac{\pi}{3} = 1$$

$$\sqrt{3} \sin x = 1$$

$$\sin x = \frac{1}{\sqrt{3}}$$

$$\sin x = \frac{\sqrt{3}}{3}$$

$$x = .615\dots$$

$$x = .615, 2.526$$

$$x = \frac{\pi}{6} + \frac{\pi}{2} n$$

$$= \frac{\pi}{3} + \frac{\pi}{2} n \quad \left. \vphantom{x = \frac{\pi}{6} + \frac{\pi}{2} n} \right\} n \in \mathbb{Z}$$

or

$$x = \frac{\pi}{6} n, n \in \mathbb{Z}$$